Problem I:

Consider the oscillation of a car after it hits a pot hole. The solution to this problem is given by:

\[ y(t) = y_0 \exp(-at) \left[ \cos(\omega t) + \left( \frac{a}{\omega} \right) \sin(\omega t) \right] \]

Where \( y \) is the vertical displacement from the equilibrium position, \( y_0 \) is the initial displacement, \( a = \frac{c}{2m} \), \( c \) = damping coefficient, \( m \) = mass, \( \omega = \sqrt{b - a^2} \), \( b = k/m \), and \( k \) is the spring constant.

Let \( c = 1.4E+04 \) (kg/sec), \( k = 1.25E+06 \) (kg/sec^2), \( m = 1.20E+03 \) (kg), and \( y_0 = -0.3 \) (m).

(i) Find the fourth time the car will pass through the equilibrium position (\( y = 0.0 \)) using (a) the bisection method, (b) false position method, (c) Newton-Raphson method, (d) Secant method. Plot for each case the approximate root versus iteration number to compare the convergence rates. Discuss the results.

(ii) Would the results change if the initial displacement was 2.0m instead of -0.3m? Explain.

(iii) How much would the first root change if the mass was altered by 20%?

Problem II:

Solve the following system of two non-linear equations

(a) Using the method of substitution
(b) using Jacobi iteration method

\[ X - 0.2XY + 1 = 0 \]
\[ -0.2Y + 0.5XY - 0.2 = 0 \]
(Note: X and Y here correspond to the equilibrium population of prey and predator, respectively, in a particular model – see Yakowitz and Szidarovszky, 1989)