The following are values for sample mean and variance for n=20 values of interior surface roughness for pipes used in oil fields.

\[
\bar{X} = 1.88 \mu m \\
S^2 = 0.27
\]

Assuming a normal distribution for the roughness on such pipes, calculate the 95% confidence interval for the theoretical mean \( \mu \).

The following is a histogram of the actual data:

(Note: This data, given the low number of samples, does not exhibit a normal distribution. For greater accuracy a T-distribution should be used; but for brevity, a normal distribution was assumed)

Solution:

\[ P(-c \leq z \leq +c) = 0.95 \]
\[ = \Phi(c) - \Phi(-c) = \Phi(c) - [1 - \Phi(c)] \]
\[ 2\Phi(c) - 1 = 0.95 \]
\[ \Phi(c) = 0.975 \]

From Table 8.1.1:

\[
\frac{1.9 - 2.0}{0.97128 - 0.97725} = \frac{c - 2.0}{0.975 - 0.97725}
\]

\[ c = 1.962 \]

Using eq(8.1.38) and eq(8.1.35)

\[
k = \frac{cS}{\sqrt{n}} = \frac{(1.962)(\sqrt{0.27})}{\sqrt{20}} = 0.228
\]

Now

\[ P(\bar{X} - k \leq \mu \leq \bar{X} + k) \]
\[ P(1.88 - 0.228 \leq \mu \leq 1.88 + 0.228) = 0.95 \]
\[ P(1.652 \leq \mu \leq 2.108) = 0.95 \]