Problem 1:

Assume a normal distribution
For a confidence interval of 0.9, \( \varphi(c) = 0.95 \). From a normal distribution table, we find the following values for \( c \):

\[
c = 1.6 \text{ for } \varphi(c) = 0.94520 \\
c = 1.7 \text{ for } \varphi(c) = 0.95543
\]

Interpolating between these two values yields \( c = 1.647 \).

Once \( c \) is known, \( k \) can be calculated as:

\[
k = \frac{cS}{\sqrt{N}} = \frac{(1.647) \times 24}{\sqrt{39}}
\]

\[k = 6.33\]

Now that \( k \) is known, the confidence interval of the mean \( \bar{X} \) is found by:

\[
P(\bar{X} - k \leq \bar{X} \leq \bar{X} + k) = 0.9 \\
P(65 - 6.33 \leq \bar{X} \leq 65 + 6.33) = 0.9 \\
P(58.67 \leq \bar{X} \leq 71.33) = 0.9
\]

The significance of this confidence interval is that, for any similar population (i.e. another class), the mean score of that sample has a 90% chance of falling within the specified range.
Problem 2:

(a) this is a 2\textsuperscript{nd} order non-linear, non-homogeneous ordinary differential equation.

- 2\textsuperscript{nd} order because of $u''$
- non-linear because of $u^3$
- ode because there is only one independent variable
- non-homogeneous because the RHS is non-zero

\[
y = u'
\]
\[
u'' = y' = -ku - u^3 + a \cos(t)
\]

(b) $u' = y$
\[
u(0) = y
\]
\[
y(1) = 0
\]

(c) To start the calculations, say using Euler method, we need to know both $u$ and $y$ at either end. If this is not the case the best thing to do is guess either $y$ at $t=0$ or $u$ at $t=1.0$. March until the other end. Check if the condition at the other end is satisfied, iterate until it matches.
Problem 3:

\[
(T^* + s) = b \frac{T^{\frac{3}{2}}}{\mu^*}
\]

\[
\frac{s}{b} + \frac{1}{b} = \frac{T^{\frac{3}{2}}}{\mu^*}
\]

\[
a_0 = \frac{s}{b}, a_1 = \frac{1}{b}
\]

\[
x = T^*, y = \frac{T^{\frac{3}{2}}}{\mu^*}
\]

<table>
<thead>
<tr>
<th>T^*</th>
<th>μ</th>
<th>x</th>
<th>y</th>
<th>x^2</th>
<th>xy</th>
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<tbody>
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<td>SUM</td>
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<td>16.32</td>
<td>120.3889</td>
<td>89.11107</td>
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Applying linear regression

\[
5a_0 + 21.33a_1 = 16.32
\]
\[
21.33a_0 + 120.39a_1 = 89.11
\]

Using Matlab to solve this system of equations yields

\[
a_0 = 0.455
\]
\[
a_1 = 0.659
\]
\[
b = \frac{1}{.659} = 1.51
\]
\[
s = (1.51) *.455 = 0.68
\]
Problem 4

(a)

Characteristic Equation

\[
det \{ [A] - \lambda [T] \} = 0
\]
\[
\begin{bmatrix}
3 & -2 & 1 \\
-2 & 4 & -1 \\
1 & -1 & 5
\end{bmatrix}
- \lambda
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} = 0
\]
\[
\begin{bmatrix}
3 - \lambda & -2 & 1 \\
-2 & 4 - \lambda & -1 \\
1 & -1 & 5 - \lambda
\end{bmatrix} = 0
\]
\[
(3 - \lambda)[(4 - \lambda)(5 - \lambda) - (-1)(-1)] - (-2)[-2(\lambda - 2) - (-1)(1)] + 1[-2(-1) - 1(4 - \lambda)] = 0
\]
\[
(3 - \lambda)[\lambda^2 - 9\lambda + 19] + 2[2\lambda - 9] + (\lambda - 2) = 0
\]
\[
-\lambda^3 + 12\lambda^2 - 41\lambda + 37 = 0
\]

(b)

Scanning

<table>
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<th>f(\lambda)</th>
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<td>-35</td>
</tr>
<tr>
<td>9</td>
<td>-89</td>
</tr>
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Plotting

The largest root is between 6 and seven
Problem 5

Solving for the exact solution yields:

\[ 2y^{1/2} = 2 - \frac{x^2}{2} \]

\[ y = \left(1 - \frac{x^2}{4}\right)^2 \]

<table>
<thead>
<tr>
<th>Euler X</th>
<th>Euler Y</th>
<th>RK2 X</th>
<th>RK2 Y</th>
<th>Exact Y</th>
<th>Etrue</th>
<th>Eapprox</th>
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