Electrical Resistance Strain Gages - Applications to Beam Bending

SUPPLEMENT On STRAIN INDICATORS and STRAIN DISTRIBUTIONS

Sensitivity of Wheatstone- Bridge Circuits

**DEFINITION** – The change in total voltage drop, ΔE, generated by a Wheatstone bridge for every unit of measured strain, ε, can be expressed as follows, by using the basic equation of such a bridge, Eq. (10):

\[
S_c = \frac{\Delta E}{\varepsilon} = \frac{V}{\varepsilon (1 + r)^2} \left[ \frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} \right]
\]  

(18)

which can be reduced to the following form for a circuit with "n" active gages (n = 1, 2, 3, or 4):

\[
S_c = V \frac{r}{(1 + r)^2} nS_g
\]

(19)

so that the sensitivity of a circuit depends on:

- The applied voltage, "V" – note that the Sensitivity Equation, (19), is applicable to circuits where "V" is constant (fixed) and independent of the current through the gage.
- The Circuit EFFICIENCY, \(r/(1+r)^2\), which is maximum when \(r = R_2/R_1 = 1\).
- The number of ACTIVE arms, "n", in the bridge
- The Gage factor, \(S_g\)

**LIMITS on Circuit Sensitivity, \(S_c\):**

- Case of Fixed VOLTAGE applied to the bridge – maximum sensitivity is achieved with FOUR active gages, when \((S_c)_{max} = S_g V \) (since \(r = 1\)), whereas the minimum value is obtained for one active arm, when \((S_c)_{min} = S_g V /4\)
- Case of Fixed CURRENT, and variable voltage – circuit sensitivity is limited by the POWER Dissipated by those specific arms in the bridge that contain active gages, which can be calculated as \(P_g = R_g I_g^2\).

**FOUR Cases of Typical Bridge Arrangements**

- **Case I** - ONE ACTIVE Gage, in position "1" – employed in dynamic and static strain measurements where temperature compensation is not critical. The remaining three "dummy" resistors are chosen so that the condition for a BALANCED bridge is satisfied, \(R_1R_3 = R_2R_4\), and the voltage supply remains within the limits of reasonable power dissipation, which can be included in the Sensitivity equation, by eliminating the voltage:
\[ V = I_g (R_1 + R_2) = I_g R_g (1 + r) = (1 + r)(P_g R_g)^{1/2}, \quad \text{so that :} \]

\[ S_c = \frac{r}{1 + r} S_g (P_g R_g)^{1/2} \quad (20) \]

which indicates that the bridge sensitivity depends on TWO factors:

- The **Gage Selection**, represented by \( S_g (P_g R_g)^{1/2} \)
- The **Circuit Efficiency**, represented by \( r/(1+r) \)

⇒ Choose "r" reasonably HIGH: \( r = 9 \) yields 90% circuit efficiency, and with a strain gage for which \( R_g = 120 \Omega \), and \( P_g = 0.15 \) W, the required voltage is \( V = 42.4 \) V.

- **Case II** – One ACTIVE gage in Arm "1", and One DUMMY gage in Arm "2", as a configuration used for **Temperature Compensation**, where \( R_1 = R_2 \), and the bridge sensitivity is given by the equation:

\[ S_c = \frac{1}{2} S_g (P_g R_g)^{1/2}, \quad \sin ce \quad V = 2I_g R_g \quad (21) \]

Note that the Circuit Efficiency is FIXED at 0.5 because of the condition \( R_1 = R_2 \)
⇒ Circuit Efficiency is REDUCED by 50%.

- **Case III** – One ACTIVE gage in the "R_1" position, and one DUMMY gage in the "R_4" position, with fixed resistors of any value placed in the positions "R_2 and R_3". This arrangement still provides temperature compensation (the resistance changes caused by temperature variations cancel out in the circuit), but the Circuit Sensitivity remains the SAME as in Case I, since:

\[ V = I_g (R_1 + R_2) = (1 + r)(P_g R_g)^{1/2}, \quad \Rightarrow \quad S_c = \frac{r}{1 + r} S_g (P_g R_g)^{1/2} \quad (22) \]

- **Case IV** - FOUR Active gages are placed in the bridge, with one gage in each of its four arms. Such a bridge is slightly MORE than TWICE as sensitive as a bridge with a single-active-arm (Cases I or III), it IS temperature compensated, but four gages are a high price to pay for the higher sensitivity. The sensitivity of such a bridge is determined as follows:

\[ V = 2I_g R_g = 2(P_g R_g)^{1/2} \quad \Rightarrow \quad S_c = 2I_g R_g S_g = 2S_g (P_g R_g)^{1/2} \]

\[ \sin ce \quad R_1 = R_2 = R_3 = R_4 = R_g \quad (23) \]

⇒ Circuit Sensitivities can be varied between \( \frac{1}{2} \) and 2 times \( S_g (P_g R_g)^{1/2} \)
Null – Balance Bridges

OBJECTIVE – More accurate and less expensive approach than direct voltage readout, $\Delta E$, to measuring the effect of the change in resistance of the active strain gage in STATIC loading \( \Rightarrow \) founded on the concept of changing the resistance of one or more arms in the bridge to match the resistance change of the active gage.

PRICIPLE of OPERATION – Figure 1.4 below:

- A Slide-Wire Resistance or Helical Potentiometer is placed across the connectors "B" and "D" of a Wheatstone bridge, such that the center tap of the potentiometer is connected to node "C" of the bridge – parallel balance configuration

- Sensitive instrument for voltage measurements is placed across the bridge between connection nodes "B" and "D".

- A single-active-gage is placed in arm "1", though active gages may be placed in any or all arms, and the bridge BALANCED, so that:

  \[
  R_1R_3 = R_2R_4 \quad \text{and} \quad R_5 = R_6
  \]

- Initially, the bridge is balanced, and no voltage is read by the voltmeter, "G". A change in the active resistance, $R_1$, upsets the initial balance, and a non-zero voltage is indicated by the meter "G".

- The potentiometer is adjusted by modifying the resistances $R_5$ and $R_6$ so that they are no longer equal to each other, until the bridge is AGAIN balanced and no voltage is shown by the meter.

- The voltage is measured only to establish the NULL-POINT, whereas the mechanical movement of the potentiometer, which is calibrated to be proportional with the resistance change of the active gage, and it is not dependent on the input voltage, "V", serves as the readout means of the system.

SENSITIVITY and RANGE of MEASUREMENTS

- Nonlinear effects limit the overall range of strain values that can measured with a Parallel-Balance bridge circuit \( \Rightarrow \) the allowable range decreases rapidly as the ratio of resistances $R_5/R_2$ increases.

- The SENSITIVITY of the Parallel-Balance circuit increases, however, when the above atio of resistances increases, since:
\[ S_{pb} = \frac{\Delta R_5 / R_5}{\varepsilon} = \frac{S_g}{2} \left(1 + \frac{R_5}{R_2}\right), \sin ce \quad \varepsilon = \frac{2}{S_g} \frac{1}{1 + R_5 / R_2} \frac{\Delta R_5}{R_5} \quad (24) \]

- The allowable change in resistance is limited to \(0<\Delta R_5/R_5<0.1\), by the nonlinear terms that are neglected in the above equations.

- Practical Configuration of Parallel-Balance arm constructed from two fixed resistors, \(R_a\) and \(R_b\), and a Potentiometer, \(R_p\) (Fig. 1.5)

**Commercial Strain Indicators**

- **NULL-BALANCE** system for STATIC strain measurements is provided by an arrangement of two bridges, an ACTIVE one for the strain gages, and a REFERENCE one, that contains either fixed or variable resistors (Fig. 1.6)

- **Initial Balance** is achieved by adjusting the resistors in the reference bridge, which are also adjusted to null out the meter reading associated with any strain measured by the gages in the active bridge.

- **Switch and Balance Unit** connected to the Strain Indicator allows successive readings of signals from up to 10 separate gages (10 different channels), without any rewiring.

- **Characteristics of P-350A Strain Indicator by Vishay Instruments:**
  
  - Powered by Oscillator that provides 1000-Hz Square Wave output with 1.5 V (rms), with no control over the voltage (not needed, since readout is independent of "V" in the null position).
  
  - **Range of Gage Resistances:** not less than 50 \(\Omega\) (overloads the oscillator), and not higher than 2000 \(\Omega\) (excessive load on the bridge circuit).
  
  - **Range of Gage Factor** for direct calibration of STRAIN readings, to \(\pm 2 \mu\text{strain}\), with an accuracy of \(\pm 0.1\%\), is: \(1.5<S_g<4.5\).
**Fig. 1.4 - Parallel Balance Circuit for the Null-Balance Wheatstone Bridge**

\[ R_5 = R_a + 0.5R_p \]
\[ R_6 = R_b + 0.5R_p \]
\[ R_a = R_b \]
\[ \Delta R_5 = +0.5R_p \]

**Fig. 1.5 - The Parallel Arm of a Null-Balance Circuit Containing Two Fixed Resistors and One Potentiometer**
Fig. 1.5 - Schematic of Reference Bridge Arrangement in a Strain Indicator
TRANSDUCER Applications of Strain Gages

OBJECTIVE – Instruments that utilize strain gages as SENSORS (or sensing devices) to measure other quantities (based on LINEAR relationships between the strain and those quantities): load, pressure, torque, displacement, and acceleration.

LOAD CELLS:

- **Tension Bar** – Four identical active gages \( r = \frac{R_2}{R_1} = 1 \) are mounted in the central section of a prismatic tension specimen:
  - Two in the LONGITUDINAL direction, on opposite faces of the bar, form the resistances \( R_1 \) and \( R_3 \) of the Wheatstone bridge.
  - Two in the TRANSVERSE direction, on opposite faces of the bar, form the resistances \( R_2 \) and \( R_4 \) of the bridge.

- **The Output/ Input Voltage Ratio** of the Wheatstone bridge is given by the basic bridge equation, Eq. (10), in the following form:

\[
\frac{\Delta E}{V} = \frac{1}{4} \left( \frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} \right) \quad (25)
\]

  - The Changes in the Resistance of the four strain gages are related to their Gage Factor in accordance with the following relationships:

\[
\frac{\Delta R_1}{R_1} = \frac{\Delta R_3}{R_3} = S_g \varepsilon_a = \frac{S_g P}{AE}, \quad \frac{\Delta R_2}{R_2} = \frac{\Delta R_4}{R_4} = S_g \varepsilon_t = -\frac{\nu S_g P}{AE} \quad (26)
\]

  - Linear relationship between the output voltage and the applied load on the tension member is obtained by substituting Eqs.(26) into Eqs. (25)

\[
\frac{\Delta E}{V} = \frac{S_g P}{2AE} (1 + \nu) \equiv \frac{P}{AE} (1 + \nu) \quad \text{when} \quad S_g \approx 2 \quad (27)
\]

  - RANGE of the Load Cell, and Maximum Measurable Stress:

\[
P_R = A \frac{\Delta E}{V} \frac{E}{1+\nu}, \quad \sigma = \frac{P}{A} = \frac{\Delta E}{V} \frac{E}{1+\nu} \quad (28)
\]

  - Common Limits are \( 0.001 < \Delta E/V < 0.003 \), and Common Material is Steel, with \( \nu = 0.3 \), and \( E = 30 \times 10^6 \text{ lb/in}^2 \) (207 Gpa).
o The Output of Tensile Load Cell is Independent of Either Bending Moments, or Applied Torque, since the output signal is equal to zero when $\varepsilon_{xx}=0$ and $\varepsilon_{zz}=0$

DIAPHRAGM PRESSURE TRANSDUCERS

o OBJECTIVE – Special-purpose strain gage is utilized as the sensor of a Diaphragm Pressure Transducer, where the strain gage is mounted on one side of the diaphragm, while the other is exposed to the pressure.

o Its design provides Maximum Output Voltage for radial and hoop (circumferential) strain components created by the applied pressure, "p". Their radial distribution is represented by the following equations, in terms of the position variable, "r", the thickness of the diaphragm, "t", its outside radius, "R_0"

$$\varepsilon_{rr} = \frac{3p(1-\nu^2)}{8Et^2}(R_0^2 - 3r^2), \quad \varepsilon_{\theta\theta} = \frac{3p(1-\nu^2)}{8Et^2}(R_0^2 - r^2) \quad (29)$$

o Circumferential Grids in the Center of the Diaphragm, since the hoop strain is maximum at the center (where $r = 0$), and it is always positive – two separate parts of the grid, connected to the arms $R_1$ and $R_3$ of the Wheatstone Bridge.

o Radial Grids Near the Edge of the Diaphragm, since the radial strain is positive in the central region, but it reaches its maximum (negative) value when $r = R_0 \Rightarrow$ two separate parts of the grid, connected to the arms $R_2$ and $R_4$ of the bridge.

o Linear Relationship between the applied pressure and the Voltage Output of the bridge is obtained by substituting into the basic bridge equation (Eq.(10)), the AVERAGED strain, or change in resistance, $\Delta R/R$, in the circumferential and the radial grids:

$$\frac{\Delta E}{V} = 0.82 \frac{pR_0^2 (1-\nu^2)}{t^2E} \quad (30)$$

o Linearity of the voltage output, $\Delta E/V$, is maintained within 0.3%, if the center deflection, $w_c$, is limited to $w_c < t/4$, where the formula of this deflection is given below:

$$w_c = \frac{3pR_0^4 (1-\nu^2)}{16t^3E} \quad (31)$$