

MAE 320- Chapter 4

Energy Analysis of Closed Systems

The content and the pictures are from the text book: Çengel, Y. A. and Boles, M. A., "Thermodynamics: An Engineering Approach," McGraw-Hill, New York, 6th Ed., 2008

Objectives

- Examine the moving **boundary work** or $P dV$ work
- Identify the first law of thermodynamics for **closed systems (fixed mass)**.
- Develop the general **energy balance applied to closed systems**.
- Define the **specific heat** at constant volume and the specific heat at constant pressure.
- **Relate the specific heats** to the calculation of the changes in **internal energy and enthalpy of ideal gases**.
- Describe incompressible substances and determine the changes in their internal energy and enthalpy.
- Solve energy balance problems for closed (fixed mass) systems that involve heat and work interactions for general **pure substances, ideal gases, and incompressible substances**.

Review of Basic Concepts

- Incompressible system
- Compressible system
- Simple system (no electric, gravity and magnetic field)
- Simple compressible system
- For a simple compressible, two independent intensive properties can determine the state (for example?)
- Closed systems (boundary can be fixed or movable)
- Equilibrium (thermal, mechanical, chemical)
- Quasi-equilibrium

Moving Boundary Work

The work associated with a moving boundary is called **boundary work**.

boundary work ($P dV$ work): The expansion and compression work in a piston-cylinder device.

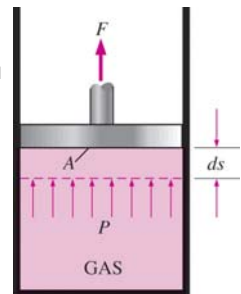
A gas does a differential amount of work δW_b , as it forces the piston to move by a differential amount ds .

$$\delta W_b = F ds = PA ds = P dV$$

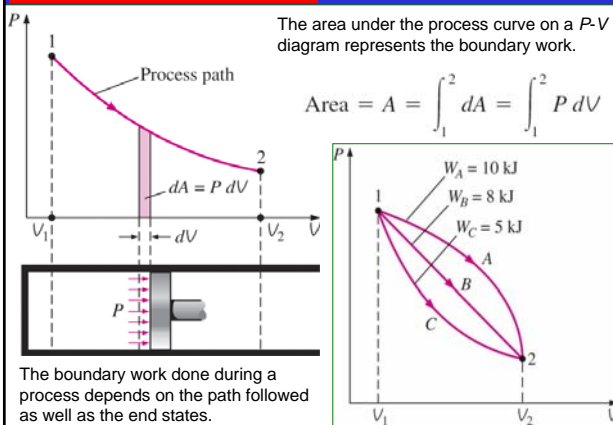
Work is a path function, use δW_b instead of ΔW_b .

$$W_b = \int_1^2 P dV \quad (\text{kJ})$$

W_b is positive \rightarrow for expansion
 W_b is negative \rightarrow for compression

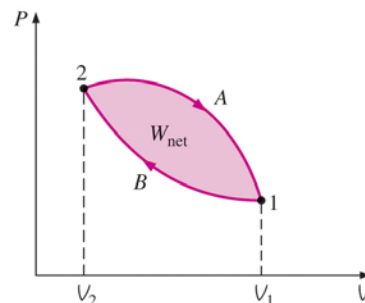


Moving Boundary Work



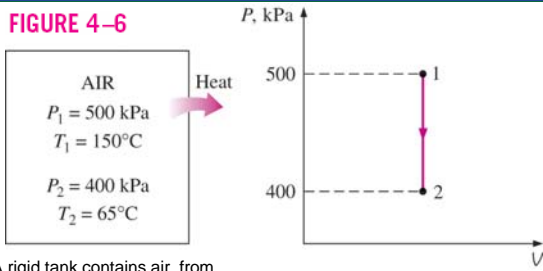
Moving Boundary Work

The net work done during a cycle is the difference between the work done by the system (Path A: From Point 2 to Point 1) and the work done on the system (Path B: From Point 1 to Point 2).



Moving Boundary Work

FIGURE 4-6



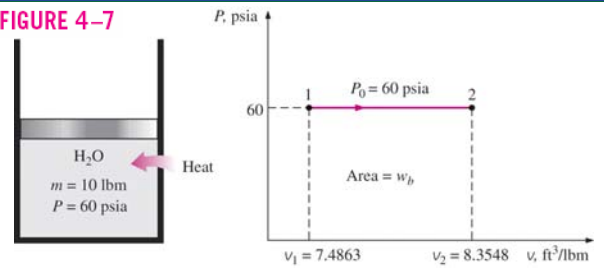
A rigid tank contains air, from State 1 ($P_1 = 500 \text{ kPa}$, $T_1 = 150^\circ\text{C}$) to State 2 ($P_2 = 400 \text{ kPa}$, $T_2 = 65^\circ\text{C}$)

Constant-volume processes

$$W_b = \int_1^2 P dV \stackrel{0}{=} 0$$

Moving Boundary Work

FIGURE 4-7



Constant-pressure processes

$$W_b = \int_1^2 P dV = P_0 \int_1^2 dV = P_0(V_2 - V_1)$$

Since $V = mv$, Hence $W_b = P_0(mv_2 - mv_1) = mP_0(v_2 - v_1)$

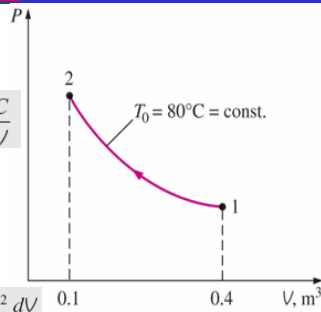
Moving Boundary Work

Isothermal processes

For an idea gas at the constant temperature

$$PV = mRT_0 = C \quad \text{or} \quad P = \frac{C}{V}$$

Where C is a constant.



$$W_b = \int_1^2 P dV = \int_1^2 \frac{C}{V} dV = C \int_1^2 \frac{dV}{V}$$

$$= C \ln \frac{V_2}{V_1} = P_1 V_1 \ln \frac{V_2}{V_1}$$

Moving Boundary Work

Polytropic processes

$PV^n = C \Rightarrow P = CV^{-n}$ n - polytropic exponent constant

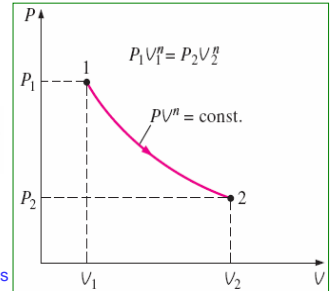
$$W_b = \int_1^2 P dV = \int_1^2 CV^{-n} dV = C \frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2 V_2 - P_1 V_1}{1-n}$$

$$W_b = \frac{mR(T_2 - T_1)}{1-n}$$

When $n = 1$ (isothermal process)

$$W_b = \int_1^2 P dV = \int_1^2 CV^{-1} dV$$

$$= PV \ln \left(\frac{V_2}{V_1} \right)$$



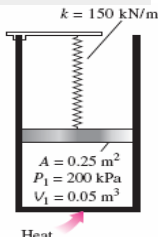
P-V diagram for a polytropic process

Moving Boundary Work

EXAMPLE 4-4

A piston-cylinder device contains 0.05 m^3 of a gas initially at 200 kPa . At this state, a linear spring that has a spring constant of 150 kN/m is touching the piston but exerting no force on it. Now heat is transferred to the gas, causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the cross-sectional area of the piston is 0.25 m^2 , determine (a) the final pressure inside the cylinder, (b) the total work done by the gas, and (c) the fraction of this work done against the spring to compress it.

Assumptions 1 The expansion process is quasi-equilibrium



Moving Boundary Work

EXAMPLE 4-4

(a) The enclosed volume at the final state is

$$V_2 = 2V_1 = (2)(0.05 \text{ m}^3) = 0.1 \text{ m}^3$$

Then the displacement of the piston (and of the spring)

$$x = \frac{\Delta V}{A} = \frac{(0.1 - 0.05) \text{ m}^3}{0.25 \text{ m}^2} = 0.2 \text{ m}$$

The force applied by the linear spring at the final state is

$$F = kx = (150 \text{ kN/m})(0.2 \text{ m}) = 30 \text{ kN}$$

The additional pressure applied by the spring on the gas

$$P = \frac{F}{A} = \frac{30 \text{ kN}}{0.25 \text{ m}^2} = 120 \text{ kPa}$$

The original pressure is 200 kPa in the absence of a spring

The total pressure:

$$200 \text{ kPa} + 120 \text{ kPa} = 320 \text{ kPa}$$

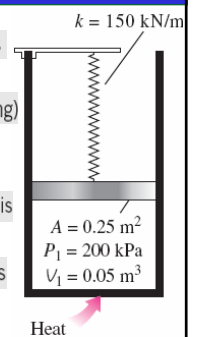


FIGURE 4-10

Moving Boundary Work

Example 4-4:

The total work done by the gas:

$$W = \int_1^2 P dV = \int_1^2 (P_o + P_{spring}) dV$$

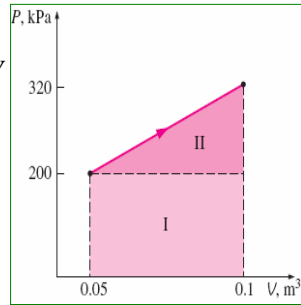
$$W = \int_1^2 P_o dV + \int_1^2 P_{spring} dV$$

$$W = P_o(V_2 - V_1) + \int_1^2 \frac{F}{A} dV$$

$$W = P_o(V_2 - V_1) + \int_1^2 \frac{kx}{A} \cdot A dx$$

$$W = P_o(V_2 - V_1) + \int_1^2 kx dx$$

$$W = P_o(V_2 - V_1) + \frac{1}{2}k(x_2^2 - x_1^2)$$



Moving Boundary Work

EXAMPLE 4-4

The total work done by the gas:

$$W = P_o(V_2 - V_1) + \frac{1}{2}k(x_2^2 - x_1^2)$$

$$W = 200 \times 10^3 \text{ Pa} \times (0.1 \text{ m}^3 - 0.05 \text{ m}^3) + \frac{1}{2} \times 150 \times 10^3 \text{ (N/m)} \times [(0.2 \text{ m})^2 - 0]$$

$$W = 10 \times 10^3 \text{ (Pa} \cdot \text{m}^3) + 3 \times 10^3 \text{ (N} \cdot \text{m)}$$

$$W = 10 \text{ kJ} + 3 \text{ kJ} \\ = 13 \text{ kJ}$$

(c) The work done against the spring is 3 kJ

Energy Balance For Closed Systems

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{system}}_{\text{Change in internal, kinetic, potential, etc., energies}} \quad (\text{kJ})$$

Energy balance for any system undergoing any process

For a closed system:

$$Q_{in} - Q_{out} + W_{in} - W_{out} = \Delta E_{system} = \Delta U + \Delta PE + \Delta KE$$

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\frac{dE_{system}}{dt}}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \quad (\text{kW})$$

Energy balance in the rate form

Energy balance in differential form:

$$\delta E_{in} - \delta E_{out} = dE_{system}$$

Energy Balance For Closed Systems

$$(Q_{in} - Q_{out}) + (W_{in} - W_{out}) + (E_{mass,in} - E_{mass,out}) = \Delta U + \Delta KE + \Delta PE$$

For a closed system without change in the KE and PE

$$(Q_{in} - Q_{out}) - (W_{out} - W_{in}) = \Delta U = U_2 - U_1$$

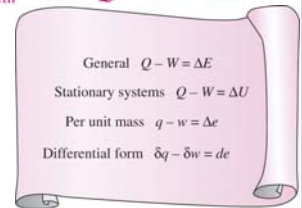
$$\text{If } Q = Q_{net,in} = Q_{in} - Q_{out}$$

$$W = W_{net,out} = W_{out} - W_{in}$$

$$Q_{net,in} - W_{net,out} = \Delta E_{system} \quad \text{or} \quad Q - W = \Delta E$$

Energy balance when sign convention is used:

- heat input, +
- heat output, -
- work output, work done by system, +
- work input, work done on a system, -



Energy Balance For Closed Systems

For a closed System:

$$e_{in} - e_{out} = \Delta e_{system} \quad (\text{kJ/kg})$$

$$q_{in} - q_{out} + w_{in} - w_{out} = \Delta e_{system} = \Delta u + \Delta pe + \Delta ke$$

The total quantities are related to the quantities per unit time is

$$Q = \dot{Q} \Delta t, \quad W = \dot{W} \Delta t, \quad \text{and} \quad \Delta E = (dE/dt) \Delta t \quad (\text{kJ})$$

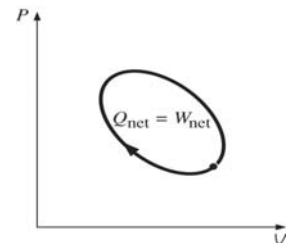
Energy Balance For Closed Systems

$$Q_{net,in} - W_{net,out} = \Delta E_{system} \quad \text{or} \quad Q - W = \Delta E$$

For a cycle $\Delta E = 0$, thus $Q = W$

For a closed system, energy balance for a cycle:

$$W_{net,out} = Q_{net,in} \quad \text{or} \quad \dot{W}_{net,out} = \dot{Q}_{net,in}$$



Energy balance for a constant-pressure expansion

Example 4-5

A piston-cylinder device contains 25 g of saturated water vapor that is maintained at a constant pressure of 300 kPa. A resistance heater within the cylinder is turned on and pass a current of 0.2 A for 5 min from a 120 V source. At the same time a heat loss of 3.7 kJ occurs. (a) show that for a closed system the boundary work W_b and the change in internal change ΔU in the first law relation can be combined to one term, ΔH , for a constant-pressure process. (b) determine the final temperature of the steam.

Analysis: The tank is a stationary closed system. Thus the $\Delta KE = 0$, $\Delta PE = 0$

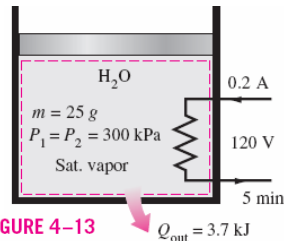


FIGURE 4-13

Energy balance for a constant-pressure expansion

Example 4-5

$$Q_{\text{net,in}} - W_{\text{net,out}} = \Delta E_{\text{system}}$$

$$Q - (W_{\text{other}} + W_b) = \Delta U = U_2 - U_1$$

$$Q - [W_{\text{other}} + P_o(V_2 - V_1)] = U_2 - U_1$$

$$Q - W_{\text{other}} - P_o(V_2 - V_1) = U_2 - U_1$$

$$Q - W_{\text{other}} = P_o V_2 - P_o V_1 + U_2 - U_1$$

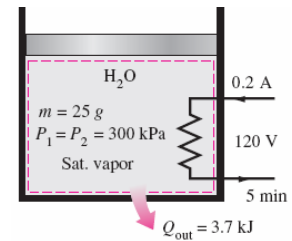
For a constant-pressure process

$$P_1 = P_2 = P_o$$

$$Q - W_{\text{other}} = (U_2 + P_2 V_2) - (U_1 + P_1 V_1)$$

$$H = U + PV$$

$$Q - W_{\text{other}} = H_2 - H_1 \quad \text{For a constant-pressure expansion or compression process}$$



Energy balance for a constant-pressure expansion

Example 4-5

$$Q - W_{\text{other}} = H_2 - H_1$$

$$Q = -3.7 \text{ kJ}$$

W_{other} is the work done on the gas by the electric heater. Hence its sign is "-"

$$W_{\text{other}} = -120 \text{ V} \times 0.2 \text{ A} \times 5 \times 60 \text{ s}$$

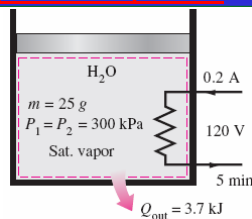
$$= -7200 \text{ J} = -7.2 \text{ kJ}$$

$$(-3.7 \text{ kJ}) - (-7.2 \text{ kJ}) = \Delta H = m(h_2 - h_1)$$

$$\text{State 1: } \left. \begin{array}{l} P_1 = 300 \text{ kPa} \\ \text{sat. vapor} \end{array} \right\} h_1 = h_g @ 300 \text{ kPa} = 2724.9 \text{ kJ/kg} \quad (\text{Table A-5})$$

$$(-3.7 \text{ kJ}) - (-7.2 \text{ kJ}) = (0.025 \text{ kg})(h_2 - 2724.9 \text{ kJ/kg})$$

$$h_2 = 2864.9 \text{ kJ/kg}$$



Energy balance for a constant-pressure expansion

Example 4-5

$$h_2 = 2864.9 \text{ kJ/kg} \quad \text{at } P = 300 \text{ kPa}$$

Based on the value of h_2 , we need to judge the state of water, Saturated water? or superheated water?

$$h_2 = 2864.9 \text{ kJ/kg} > h_g = 2724.9 \text{ kJ/kg} \quad (\text{Table A-6})$$

Hence it is superheated water

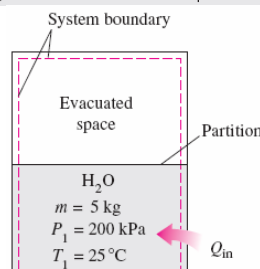
From Table A-6:

$$\text{State 2: } \left. \begin{array}{l} P_2 = 300 \text{ kPa} \\ h_2 = 2864.9 \text{ kJ/kg} \end{array} \right\} T_2 = 200^\circ \text{C}$$

Energy Balance For Closed Systems

Example 4-6

A rigid tank is divided into two equal parts by a partition. Initially, one side of the tank contains 5 kg of water at 200 kPa and 25°C, and the other side is evacuated. The partition is then removed, and the water expands into the entire tank. The water is allowed to exchange heat with its surroundings until the temperature in the tank returns to the initial value of 25°C. Determine (a) the volume of the tank, (b) the final pressure, and (c) the heat transfer for this process.



Energy Balance For Closed Systems

Example 4-6

(a) At first, determine the state of water in the initial state:

$$25^\circ \text{C} \ll T_{\text{sat}@200 \text{ kPa}} = 120.21^\circ \text{C} \quad (\text{Table A-5})$$

$$\text{Pressure } 200 \text{ kPa} > P_{\text{sat}@25^\circ \text{C}} = 3.1698 \text{ kPa} \quad (\text{Table A-4})$$

Thus in the initial state, it is a compressed liquid

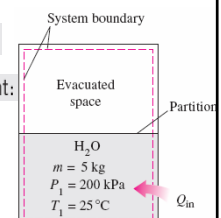
$$v_1 \cong v_f @ 25^\circ \text{C} = 0.001003 \text{ m}^3/\text{kg} \cong 0.001 \text{ m}^3/\text{kg}$$

Then the initial volume of the water is

$$V_1 = m v_1 = (5 \text{ kg})(0.001 \text{ m}^3/\text{kg}) = 0.005 \text{ m}^3$$

The total volume of the tank is twice this amount:

$$V_{\text{tank}} = (2)(0.005 \text{ m}^3) = 0.01 \text{ m}^3$$



Energy Balance For Closed Systems

Example 4-6

(b) At the final state, the specific volume of the water is

$$v_2 = \frac{V_2}{m} = \frac{0.01 \text{ m}^3}{5 \text{ kg}} = 0.002 \text{ m}^3/\text{kg}$$

Determine the state of water in the final state ($T=25^\circ\text{C}$ and v_2)

At 25°C : $v_f = 0.001003 \text{ m}^3/\text{kg}$ and $v_g = 43.340 \text{ m}^3/\text{kg}$ (Table A-4)

Since $v_f < v_2 < v_g$, the water is a saturated liquid-vapor mixture

The final pressure inside the tank:

$$P_2 = P_{\text{sat}} @ 25^\circ\text{C} = \mathbf{3.1698 \text{ kPa}} \quad (\text{Table A-4})$$

Energy Balance For Closed Systems

Example 4-6

$$Q_{\text{net,in}} - W_{\text{net,out}} = \Delta E_{\text{system}}$$

$$Q = \Delta U = m(u_2 - u_1)$$

In the initial state, it is a compressed liquid

$$u_1 \cong u_f @ 25^\circ\text{C} = 104.83 \text{ kJ/kg}$$

In the final state, it is liquid-gas mixture:

$$\text{The quality at the final state is determined from}$$

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.002 - 0.001}{43.34 - 0.001} = 2.3 \times 10^{-5}$$

$$u_2 = u_f + x_2 u_{fg} = 104.83 \text{ kJ/kg} + (2.3 \times 10^{-5})(2304.3 \text{ kJ/kg})$$

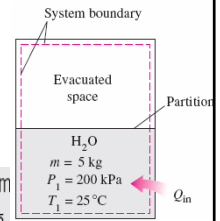
$$= 104.88 \text{ kJ/kg}$$

Substituting yields

$$Q = (5 \text{ kg})[(104.88 - 104.83) \text{ kJ/kg}] = \mathbf{0.25 \text{ kJ}}$$

Heater transfer in or out?

Positive sign means heat input



Energy Balance For Closed Systems

For a **closed system**:

$$Q_{\text{in}} - Q_{\text{out}} + W_{\text{in}} - W_{\text{out}} = \Delta E_{\text{system}} = \Delta U + \Delta PE + \Delta KE \quad (\text{kJ})$$

$$q_{\text{in}} - q_{\text{out}} + w_{\text{in}} - w_{\text{out}} = \Delta e_{\text{system}} = \Delta u + \Delta pe + \Delta ke \quad (\text{kJ/kg})$$

$$e_{\text{in}} - e_{\text{out}} = \Delta e_{\text{system}} \quad (\text{kJ/kg})$$

For a **closed system** without change in "pe" and "ke"

$$q_{\text{in}} - q_{\text{out}} + w_{\text{in}} - w_{\text{out}} = \Delta e_{\text{system}} = \Delta u = (u_2 - u_1) \quad (\text{kJ/kg})$$

$$Q_{\text{in}} - Q_{\text{out}} + W_{\text{in}} - W_{\text{out}} = \Delta E_{\text{system}} = \Delta U = m(u_2 - u_1) \quad (\text{kJ})$$

For a **closed system** without change in pe and ke, and no phase change [For example, the ideal gas (or a liquid, or a solid) in a closed system]

$$q_{\text{in}} - q_{\text{out}} + w_{\text{in}} - w_{\text{out}} = \Delta e_{\text{system}} = \Delta u = c_v(T_2 - T_1) \quad (\text{kJ/kg})$$

$$Q_{\text{in}} - Q_{\text{out}} + W_{\text{in}} - W_{\text{out}} = \Delta E_{\text{system}} = \Delta U = mc_v(T_2 - T_1) \quad (\text{kJ})$$

Attention: "**KJ**" is used as the unit of energy in the textbook.

"**J**" is used as the unit of energy in the SI unit system.

Specific Heats

Specific heat at constant volume,

c_v : The energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.

$$c_v = \left(\frac{\partial u}{\partial T} \right)_v$$

= the change in internal energy with temperature at constant volume

Specific heat at constant pressure,

c_p : The energy required to raise the temperature of the unit mass of a substance by one degree as the pressure is maintained constant.

$$c_p = \left(\frac{\partial h}{\partial T} \right)_p$$

= the change in enthalpy with temperature at constant pressure

Specific Heats

The unit of specific heat is $(\text{kJ/kg} \cdot \text{K})$ or $(\text{kJ/kg} \cdot ^\circ\text{C})$

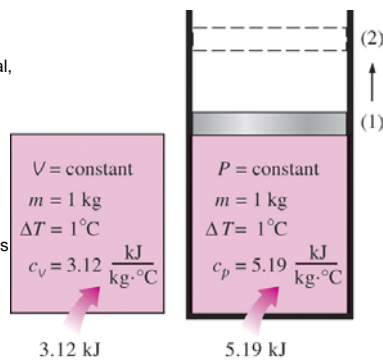
These two units are identical, since $\Delta T(\text{K}) = \Delta T(^{\circ}\text{C})$

c_v and c_p can be expressed on the molar basis With the unit:

$(\text{kJ/kmol} \cdot \text{K})$ or $(\text{kJ/kmol} \cdot ^\circ\text{C})$

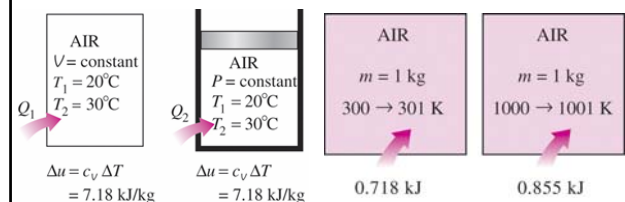
$$\bar{c}_v = c_v \cdot M$$

$$\bar{c}_p = c_p \cdot M$$



Specific Heats

- c_v and c_p are properties. They are intensive properties.
- c_v and c_p are applied to *any* substance undergoing *any* process.
- c_v is related to the changes in *internal energy* and c_p to the changes in *enthalpy*.
- c_v and c_p are temperature-dependent.
- c_p is always greater than c_v .
- A common unit for specific heats is $\text{kJ/kg} \cdot ^\circ\text{C}$ or $\text{kJ/kg} \cdot \text{K}$.



The relation $\Delta u = c_v \Delta T$ is valid for *any* kind of process, constant-volume or not.

The specific heat of a substance depends on temperature.

Specific Heats of Ideal Gases

For a gas, the internal energy is a function of temperature only, not a function of pressure or specific volume

$$u = u(T)$$

For an idea gas, $Pv = RT$

$$\left. \begin{aligned} h &= u + Pv \\ Pv &= RT \end{aligned} \right\} h = u + RT$$

$$du = c_v(T) dT$$

$$dh = c_p(T) dT$$

$$\Delta u = u_2 - u_1 = \int_1^2 c_v(T) dT$$

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T) dT$$

$$\begin{aligned} u &= u(T) \\ h &= h(T) \\ c_v &= c_v(T) \\ c_p &= c_p(T) \end{aligned}$$

For ideal gases, u , h , c_v and c_p vary with temperature only. They are independent on pressure

Specific Heat Relations of Ideal Gases

For the idea gas:

$$\left. \begin{aligned} h &= \bar{u} + RT \\ dh &= d\bar{u} + R dT \\ dh &= c_p dT \text{ and } d\bar{u} = c_v dT \end{aligned} \right\} \begin{aligned} &\rightarrow c_p = c_v + R \quad (\text{kJ/kg} \cdot \text{K}) \\ &\text{On a molar basis} \\ &\bar{c}_p = \bar{c}_v + R_u \quad (\text{kJ/kmol} \cdot \text{K}) \end{aligned}$$

$$\text{Where } \bar{c}_v = c_v \cdot M$$

$$R_u = R \cdot M$$

$$\text{Specific heat ratio } k = \frac{c_p}{c_v}$$

- The specific ratio varies with temperature, but this variation is very mild.
- For monatomic gases (helium, argon, etc.), its value is essentially constant at 1.667.
- Many diatomic gases, including air, have a specific heat ratio of about 1.4 at room temperature.

Average Specific Heat

Internal energy and enthalpy change when specific heat is taken constant at an average value

$$u_2 - u_1 = c_{v,avg}(T_2 - T_1) \quad (\text{kJ/kg})$$

$$h_2 - h_1 = c_{p,avg}(T_2 - T_1)$$

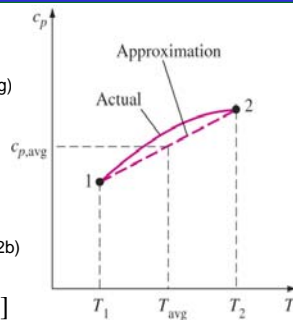
How to get the average specific heat

Way 1: $T_{avg} = \frac{1}{2}(T_1 + T_2)$

$$c_{v,avg} = c_v(T_{avg}) \quad (\text{Table A-2b})$$

Way 2: $c_{v,avg} = \frac{1}{2}[c_v(T_1) + c_v(T_2)]$

Way 3: $c_{v,avg} = \frac{u(T_2) - u(T_1)}{T_2 - T_1}$



For small temperature intervals, the specific heats may be assumed to vary linearly with temperature.

Calculation of internal energy

Calculate Δu and Δh in three ways

- By using the tabulated u and h data. This is the easiest and **most accurate** way when tables are readily available.
- By using the c_v or c_p relations (Table A-2c) as a function of temperature and performing the integrations. This is very inconvenient for hand calculations but quite desirable for computerized calculations. The results obtained are **very accurate**.
- By using average specific heats. This is very simple and certainly very convenient when property tables are not available. The results obtained are **reasonably accurate** if the temperature interval is not very large.

$$\Delta u = u_2 - u_1 \quad (\text{table})$$

$$\Delta u = \int_1^2 c_v(T) dT$$

$$\Delta u \approx c_{v,avg} \Delta T$$

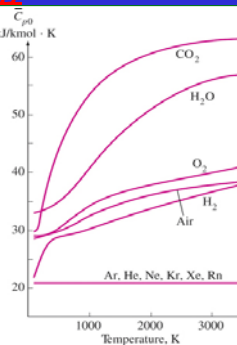
Three ways of calculating Δu .

Calculation of internal energy

- At low pressures, all real gases approach ideal-gas behavior, and therefore their specific heats depend on temperature only.
- The specific heats of real gases at low pressures are called *ideal-gas specific heats*, or *zero-pressure specific heats*, and are often denoted c_{p0} and c_{v0} .
- u and h data for a number of gases have been tabulated.

Table A-17, idea-gas properties of air

AIR		
T, K	u, kJ/kg	h, kJ/kg
0	0	0
300	214.07	300.19
310	221.25	310.24



c_p for some gases (see Table A-2c for equations).

In the preparation of ideal-gas tables, 0 K is chosen as the reference temperature.

Calculation of internal energy

Example 4-7

Air at 300 K and 200 kPa is heated at constant pressure to 600 K. Determine the change in internal energy of air per unit mass, using (a) data from the air table (Table A-17), (b) the functional form of the specific heat (Table A-2c), and (c) the average specific heat value (Table A-2b).

Calculation of internal energy

Example 4-7

(a) One way of determining the change in internal energy of air is to read the u values at T_1 and T_2 from Table A-17 and take the difference:

$$u_1 = u_{@ 300 \text{ K}} = 214.07 \text{ kJ/kg}$$

$$u_2 = u_{@ 600 \text{ K}} = 434.78 \text{ kJ/kg}$$

$$\Delta u = u_2 - u_1 = (434.78 - 214.07) \text{ kJ/kg} \\ = \mathbf{220.71 \text{ kJ/kg}}$$

Calculation of internal energy

Example 4-7

(b) The $\bar{c}_p(T)$ of air is given in Table A-2c in the form of a third-degree polynomial expressed as

$$\bar{c}_p(T) = a + bT + cT^2 + dT^3$$

where $a = 28.11$, $b = 0.1967 \times 10^{-2}$, $c = 0.4802 \times 10^{-5}$, and $d = -1.966 \times 10^{-9}$. From Eq. 4-30,

$$\bar{c}_v(T) = \bar{c}_p - R_u = (a - R_u) + bT + cT^2 + dT^3$$

$$\Delta \bar{u} = \int_1^2 \bar{c}_v(T) dT = \int_{T_1}^{T_2} [(a - R_u) + bT + cT^2 + dT^3] dT$$

$$\Delta \bar{u} = 6447 \text{ kJ/kmol}$$

$$\Delta u = \frac{\Delta \bar{u}}{M} = \frac{6447 \text{ kJ/kmol}}{28.97 \text{ kg/kmol}} = \mathbf{222.5 \text{ kJ/kg}}$$

Note: M is obtained from Table A-1

Calculation of internal energy

Example 4-7

(c) The average value of the constant-volume specific heat $c_{v,avg}$ is determined from Table A-2b at the average temperature of $(T_1 + T_2)/2 = 450 \text{ K}$ to be

$$c_{v,avg} = c_v @ 450 \text{ K} = 0.733 \text{ kJ/kg} \cdot \text{K}$$

$$\Delta u = c_{v,avg}(T_2 - T_1) = (0.733 \text{ kJ/kg} \cdot \text{K})[(600 - 300)\text{K}] \\ = \mathbf{220 \text{ kJ/kg}}$$

Note: $c_{v,avg}$ can be obtained from an alternative way:

$$c_{v,avg} = \frac{1}{2}[c_v(T_1) + c_v(T_2)]$$

$$c_v(T_1) = 0.718 \text{ kJ/kg} \quad \text{at } T_1 = 300 \text{ K}$$

$$c_v(T_2) = 0.764 \text{ kJ/kg} \quad \text{at } T_2 = 600 \text{ K}$$

$$c_{v,avg} = \frac{1}{2}[0.718 + 0.764] \text{ kJ/kg} \cdot \text{K} = 0.736 \text{ kJ/kg} \cdot \text{K}$$

Calculation of internal energy

Example 4-8

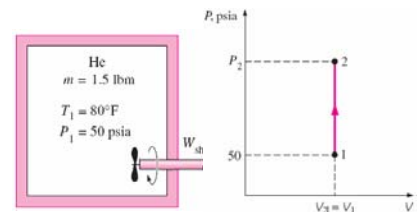
An insulated rigid tank initially contains 1.5 lbm of helium at 80°F and 50 psia. A paddle wheel with a power rating of 0.02 hp is operated within the tank for 30 min. Determine (a) the final temperature and (b) the final pressure of the helium gas.

$$1 \text{ lbm} = 0.45359 \text{ kg}$$

$$T (^{\circ}\text{C}) = [T(^{\circ}\text{F}) - 32] \cdot 5/9$$

$$1 \text{ psi} = 6.895 \times 10^3 \text{ Pa}$$

$$1 \text{ hp} = 754.7 \text{ W}$$



Calculation of internal energy

Example 4-8

For a closed system without change in KE and PE, the energy conservation equation is:

$$Q_{in} - Q_{out} + W_{in} - W_{out} = \Delta E_{system} = \Delta U = m(u_2 - u_1) = mc_{v,avg}(T_2 - T_1)$$

$$Q_{in} = 0 \quad Q_{out} = 0 \quad W_{out} = 0$$

$$W_{sh} = \Delta U = mc_{v,avg}(T_2 - T_1)$$

$$W_{sh} = \dot{W} \Delta t = (0.02 \times 754.7 \text{ W})(0.5 \times 3600 \text{ s}) = 27169.2 \text{ J} = 27.17 \text{ kJ}$$

From Table A-2(a), $C_v = 3.1156 \text{ kJ/kg} \cdot \text{K}$

$$27.17 \text{ kJ} = (1.5 \times 0.45359 \text{ kg})(3.1156 \text{ kJ/kg} \cdot \text{K})[T_2 - \frac{5}{9}(80 - 32)^{\circ}\text{C}]$$

$$T_2 = \mathbf{44.48 ^{\circ}\text{C}}$$

Calculation of internal energy

Example 4-8

(b) The final pressure is determined from the ideal-gas relation

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{50 \times 6.895 \text{ kPa}}{(273 + 31.67) \text{ K}} = \frac{P_2}{(273 + 44.48) \text{ K}}$$

$$P_2 = 359 \text{ kPa} = \mathbf{52.1 \text{ psi}}$$

Calculation of internal energy

EXAMPLE 4-10 Heating of a Gas at Constant Pressure

A piston-cylinder device initially contains air at 150 kPa and 27°C. At this state, the piston is resting on a pair of stops, as shown in Fig. 4-32, and the enclosed volume is 400 L. The mass of the piston is such that a 350-kPa pressure is required to move it. The air is now heated until its volume has doubled. Determine (a) the final temperature, (b) the work done by the air, and (c) the total heat transferred to the air.

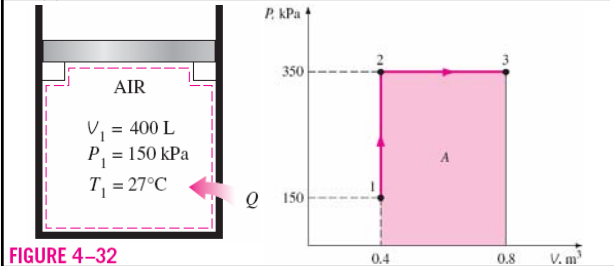


FIGURE 4-32

Calculation of internal energy

Example 4-10

(a) Air is an idea gas at high temperature and low pressure

$P_1 = 150 \text{ kPa}$ $V_1 = 400 \text{ L}$ $T_1 = 300 \text{ K}$	→	$P_2 = 350 \text{ kPa}$ $V_2 = 800 \text{ L}$ $T_2 = ?$
---	---	---

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \rightarrow \frac{(150 \text{ kPa})(V_1)}{300 \text{ K}} = \frac{(350 \text{ kPa})(2V_1)}{T_2}$$

$$T_2 = 1400 \text{ K}$$

Calculation of internal energy

Example 4-10

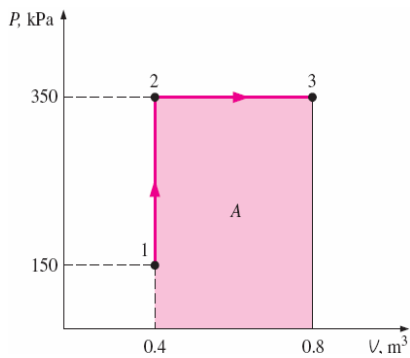
(b) The work done by gas is illustrated in the figure. The area is The amount of work done:

$$A = (V_2 - V_1)P_2$$

$$= (0.4 \text{ m}^3)(350 \text{ kPa})$$

$$= 140 \text{ m}^3 \cdot \text{kPa}$$

Hence, the work is 140 kJ



Calculation of internal energy

Example 4-10

(c) This is a stationary closed system (no mass change), and $\Delta KE=0$, $\Delta PE=0$. No other work like electric and shaft involved.

$$Q_{in} - Q_{out} + W_{in} - W_{out} = \Delta E_{system} = \Delta U = m(u_2 - u_1)$$

$$Q_{in} - W_{b,out} = m(u_2 - u_1)$$

The mass of the system can be determined from the ideal-gas relation:

R is obtained from Table A-1

$$m = \frac{P_1 V_1}{RT_1} = \frac{(150 \text{ kPa})(0.4 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(300 \text{ K})} = 0.697 \text{ kg}$$

u of air can be obtained from Table A-17

$$u_1 = u_{@ 300 \text{ K}} = 214.07 \text{ kJ/kg}$$

$$u_3 = u_{@ 1400 \text{ K}} = 1113.52 \text{ kJ/kg}$$

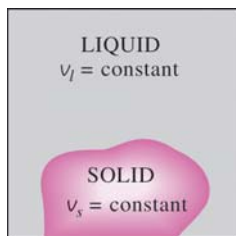
$$Q_{in} - 140 \text{ kJ} = (0.697 \text{ kg})[(1113.52 - 214.07) \text{ kJ/kg}]$$

$$Q_{in} = 767 \text{ kJ}$$

Internal Energy & Enthalpy of Solids and Liquids

Incompressible substance: A substance whose specific volume (or density) is constant.

Solids and liquids are incompressible substances.



The specific volumes of incompressible substances remain constant during a process.



The c_v and c_p values of incompressible substances are identical and are denoted by c .

Internal Energy of Solids and Liquids

For liquids and solid, the constant-volume and constant pressure processes are almost identical, Hence:

$$c_p = c_v = c$$

The internal energy changes of liquids and solids:

$$du = c_v dT = c(T) dT$$

$$\Delta u = u_2 - u_1 = \int_1^2 c(T) dT \quad (\text{kJ/kg})$$

$$\Delta u \cong c_{avg}(T_2 - T_1) \quad (\text{kJ/kg})$$

Enthalpy of Solids and Liquids

Enthalpy Changes of incompressible substances:

$$h = u + Pv$$

$$dh = du + v dP + P d\overset{0}{v} = du + v dP$$

Volume is constant for solids and liquids

$$\Delta h = \Delta u + v \Delta P \cong c_{\text{avg}} \Delta T + v \Delta P \quad (\text{kJ/kg})$$

For *solids*, the term $v \Delta P$ is insignificant and thus $\Delta h = \Delta u \cong c_{\text{avg}} \Delta T$. For *liquids*, two special cases are commonly encountered:

1. *Constant-pressure processes*, as in heaters ($\Delta P = 0$): $\Delta h = \Delta u \cong c_{\text{avg}} \Delta T$
2. *Constant-temperature processes*, as in pumps ($\Delta T = 0$): $\Delta h = v \Delta P$

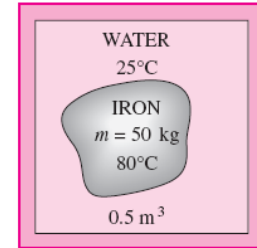
The enthalpy of a compressed liquid

$$h_{@P,T} \cong h_{f@T} + v_{f@T}(P - P_{\text{sat}@T})$$

Calculation of internal energy

EXAMPLE 4-12 Cooling of an Iron Block by Water

A 50-kg iron block at 80°C is dropped into an insulated tank that contains 0.5 m³ of liquid water at 25°C. Determine the temperature when thermal equilibrium is reached.



Calculation of internal energy

Example 4-12

Solution An iron block is dropped into water in an insulated tank. The final temperature when thermal equilibrium is reached is to be determined.

Assumptions 1 Both water and the iron block are incompressible substances. 2 Constant specific heats at room temperature can be used for water and the iron. 3 The system is stationary and thus the kinetic and potential energy changes are zero, $\Delta KE = \Delta PE = 0$ and $\Delta E = \Delta U$. 4 There are no electrical, shaft, or other forms of work involved. 5 The system is well-insulated and thus there is no heat transfer.



Calculation of internal energy

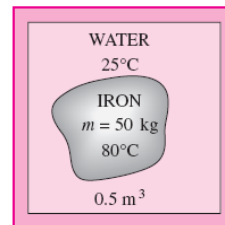
Example 4-12

Analysis: (1) we tank the whole tank as the system. (2) This is a stationary closed system. (3) The boundary of the system is fixed. (4) No any heat is transferred though the system boundary (5) no other forms of works is done. Thus energy balance on the system:

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$$

$$0 = \Delta U$$

$$\Delta U_{\text{sys}} = \Delta U_{\text{iron}} + \Delta U_{\text{water}} = 0$$



Calculation of internal energy

Example 4-12

$$[mc(T_2 - T_1)]_{\text{iron}} + [mc(T_2 - T_1)]_{\text{water}} = 0$$

The specific volume of liquid water at or about room temperature can be taken to be 0.001 m³/kg. Then the mass of the water is

$$m_{\text{water}} = \frac{V}{v} = \frac{0.5 \text{ m}^3}{0.001 \text{ m}^3/\text{kg}} = 500 \text{ kg}$$

$$c_{\text{iron}} = 0.45 \text{ kJ/kg} \cdot ^\circ\text{C} \text{ and } c_{\text{water}} = 4.18 \text{ kJ/kg} \cdot ^\circ\text{C} \text{ from Table A-3}$$

$$(50 \text{ kg})(0.45 \text{ kJ/kg} \cdot ^\circ\text{C})(T_2 - 80^\circ\text{C}) + (500 \text{ kg})(4.18 \text{ kJ/kg} \cdot ^\circ\text{C})(T_2 - 25^\circ\text{C}) = 0$$

$$T_2 = 25.6^\circ\text{C}$$

Summary

- Moving boundary work
 - ✓ W_b for an isothermal process
 - ✓ W_b for a constant-pressure process
 - ✓ W_b for a polytropic process
- Energy balance for closed systems
 - ✓ Energy balance for a constant-pressure expansion or compression process
- Specific heats
 - ✓ Constant-pressure specific heat, c_p
 - ✓ Constant-volume specific heat, c_v
- Internal energy, enthalpy, and specific heats of ideal gases
 - ✓ Specific heat relations of ideal gases
- Internal energy, enthalpy, and specific heats of incompressible substances (solids and liquids)