1. Multiple choice questions (30 points)

(1a). (3p) A process can proceed if it…

(A) Satisfy both the 1st and 2nd Laws of Thermodynamics.
(B) Violates both 1st and 2nd Law of Thermodynamics.
(C) Satisfy the 1st law but violates the 2nd Law of Thermodynamics.
(D) Satisfy the 2nd law but violates the 1st Law of Thermodynamics.
(E) It depends…

(1b). (3p) The following phrase(s) is(are) valid regarding the Kelvin-Planck statement of the 2nd Law …

(A) It is possible for a device to operate on a cycle to receive heat from a single reservoir and produce a net amount of work.
(B) Thermal efficiency of a heat engine could reach 100% if no friction is involved.
(C) A net positive work can be output by a device that adsorbs energy from a high temperature reservoir without ejecting energy to a low temperature reservoir.
(D) Work can always be converted to heat directly and completely, and the reverse is true.
(E) If the thermal efficiency of a device is less than 100%, it must be an irreversible device.
(F) It is necessary to have a low temperature reservoir to reject waste heat energy for a heat engine operating on a cycle.

(1c). (3p) The following phrase is valid regarding the Clausius statement of the 2nd Law of Thermodynamics.

(A) It is possible for any device that operates on a cycle to receive heat from a single reservoir and produce a net amount of work.
(B) Energy input is required to transfer heat energy from a low temperature reservoir to a high temperature reservoir.
(C) A refrigerator is a not cyclic device.
(D) Energy can be transferred from a low temperature reservoir to a high temperature reservoir and produce a net positive work out.
(E) Any device that violates the Kelvin–Planck statement does not always violate the Clausius statement.

(1d). (3p) Which is not an application of the 2nd Law of Thermodynamics?

(A) Predicting the direction of a process.
(B) Quantify the power produced by a device.
(C) Asserting that energy has quality.
(D) Determining the best theoretical performance of cycles, engines, and other devices.
(E) Predicting the degree of completion of chemical reactions.

(1e). (3p) The statement that the efficiency of a reversible heat engine cycle is 100% violates:

(A) The 1st Law of Thermodynamics.
(B) Both the 1st and the 2nd Laws of Thermodynamics.
(C) The Kelvin-Planck statement of the 2\textsuperscript{nd} Law of Thermodynamics.
(D) The Clausius statement of the 2\textsuperscript{nd} Law of Thermodynamics.
(E) The Carnot Principle.

(1f). (3p) The thermal efficiency of an actual power cycle A and a reversible power cycle B works under same hot (H) and cool reservoir (C) are $\eta_A$ and $\eta_B$, respectively. Please circle the one satisfy the Carnot Principle:

(A) $\eta_A > \eta_B$.
(B) $\eta_A = \eta_B$.
(C) $\eta_A < \eta_B$.
(D) Cannot be decided based on the information provided.

(1g). (3p) Please circle the valid statement(s):

(A) For an internally reversible process, no irreversibilities occur within the boundaries of the system during the process, and no irreversibilities occur outside the system boundaries.
(B) **Reversible processes deliver the most and consume the least work.**
(C) A process is called reversible if the system and all parts of its surroundings cannot be exactly restored to their respective initial states after the process has occurred.
(D) **In irreversible processes, the surroundings usually do some work on the system and therefore does not return to their original state.**
(E) During a cycle, a system can be restored to its initial state following a process, regardless of whether the process is reversible or irreversible.

(1h). (3p) A heat engine that creates energy is known as:

(A) A perpetual machine of the zeroth kind.
(B) **A perpetual machine of the first kind.**
(C) A perpetual machine of the second kind.
(D) A perpetual machine of the third kind.
(E) None of the above.

(1i). (3p) Please circle the source(s) of irreversibility(ies) among the following items.

(A) **Mixing of two fluids.**
(B) **Friction.**
(C) Frictionless pendulum.
(D) **Heat transfer across a finite temperature difference.**
(E) **Unrestrained expansion of gas at high pressure.**

(1j). (3p) Please circle the valid statement(s):

(A) **The Carnot heat engine is the most efficient of all heat engines operating between the same high- and low-temperature reservoirs.**
(B) The efficiency of reversible heat engines is independent of the working fluid and its property
(C) All reversible heat engines operating between the same two reservoirs have the same efficiency.
(D) All irreversible heat engines operating between the same two reservoirs must have the same efficiency.
(E) No heat engine can have a higher efficiency than a reversible heat engine operating between the same high- and low-temperature reservoirs.

(F) A Carnot engine is a reversible engine.

2. (12 points) A power cycle operates between two reservoirs at 700 K and 300 K, respectively. \( Q_H \) is the heat (thermal energy) received from the hot reservoir. \( Q_L \) is the heat rejected to the cool reservoir. \( W_{\text{mech}} \) is the mechanical work produced. Please determine if the following cycles are possible and explain why.

Note: you need to clearly identify the thermodynamic law violated if the cycle is impossible.

**Analysis:** For this kind of question, you need check if it satisfies both 1\(^{\text{st}}\) law (conservation of energy: \( Q_H = W_{\text{mech}} + Q_L \)) and the 2\(^{\text{nd}}\) law \( \eta_{\text{th,act}} \leq \eta_{\text{th,rev,cycle}} \). The maximum efficiency of heat engine operating with these two reservoirs can be calculated:

\[
\eta_{\text{max}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300K}{700K} = 57.14\%.
\]

(A). (3p) \( Q_H=700 \text{ kJ}, W_{\text{mech}}=400 \text{ kJ}, Q_L=300 \text{ kJ}; \)

**Solution:** \( W_{\text{mech}} + Q_L = 400kJ + 300kJ = 700kJ = Q_H \), 1\(^{\text{st}}\) Law is satisfied. \[ \eta = 1 - \frac{Q_L}{Q_H} = 1 - \frac{300kJ}{700kJ} = 57.14\% = \eta_{\text{max}} \] 2\(^{\text{nd}}\) Law is satisfied. This engine is operated on reversible cycle.

(B). (3p) \( Q_H=700 \text{ kJ}, W_{\text{mech}}=500 \text{ kJ}, Q_L=200 \text{ kJ}; \)

**Solution:** \( W_{\text{mech}} + Q_L = 500kJ + 200kJ = 700kJ = Q_H \), 1\(^{\text{st}}\) Law is satisfied. \[ \eta = 1 - \frac{Q_L}{Q_H} = 1 - \frac{200kJ}{700kJ} = 71.42\% > 57.14\% = \eta_{\text{max}} \] 2\(^{\text{nd}}\) Law is not satisfied. It is impossible for this engine to produce 500 kJ work with the consumption of 700 kJ thermal energy from hot reservoir.

(C). (3p) \( Q_H=700 \text{ kJ}, W_{\text{mech}}=300 \text{ kJ}, Q_L=600 \text{ kJ}; \)

**Solution:** \( W_{\text{mech}} + Q_L = 300kJ + 600kJ = 900kJ \neq Q_H \), 1\(^{\text{st}}\) Law is not satisfied. The operation of this engine under the described conditions is impossible. There is no need the check if 2\(^{\text{nd}}\) Law is satisfied.

(D). (3p) \( Q_H=700 \text{ kJ}, W_{\text{mech}}=600 \text{ kJ}, Q_L=100 \text{ kJ}; \)

**Solution:** \( W_{\text{mech}} + Q_L = 500kJ + 200kJ = 700kJ = Q_H \), 1\(^{\text{st}}\) law is satisfied. \[ \eta = 1 - \frac{Q_L}{Q_H} = 1 - \frac{100kJ}{700kJ} = 85.71\% > \eta_{\text{max}} = 57.14\% \] 2\(^{\text{nd}}\) law is not satisfied. It is impossible for this engine to produce 600 kJ work with the consumption of 700 kJ thermal energy from hot reservoir.

3. (6 points) A refrigerator with a coefficient of performance of 2.5 consumes 18 kW of electricity when running. Determine the rate of heat ejected into the kitchen by this refrigerator.

**Solution:** \( COP_R = \frac{Q_L}{W_{\text{in}}} \Rightarrow Q_L = COP_R \times W_{\text{in}} \Rightarrow \dot{Q}_H = \dot{Q}_L + \dot{W}_{\text{in}} = \dot{W}_{\text{in}} (COP_R + 1) = 18 kW (2.5 + 1) = 63 kW. \)
4. (7 points) A heat engine receives heat from a source at 900 °C and rejects the waste heat to a sink at 15 °C. If heat is rejected from this engine at a rate of 30 kJ/s, determine the maximum power produced by this heat engine.

**Solution:** When the heat engine is operated in a reversible process, it will achieve the maximum power. For a reversible heat engine: 
\[
\frac{Q_H}{Q_L} = \frac{T_H}{T_L} \quad \Rightarrow \quad Q_H = Q_L \cdot \frac{T_H}{T_L}.
\]
According to the 1st Law of Thermodynamics:
\[
W_{out} = Q_H - Q_L = (\frac{T_H}{T_L} - 1) \cdot Q_L = (\frac{900 + 273.15}{15 + 273.15} - 1) \times 30 \text{kJ/s} = 92.8 \text{kW}.
\]
5. \( (10 \text{ points}) \) Assume that WVU’s Engineering Sciences Building (ESB) is always maintained at 27 °C, while the outside temperature today is 12 °C. The building is heated by means of a heat pump of the coefficient of performance (COP) \( \gamma = 3.5 \), which provides the energy at an average rate of 100 kW. The electricity costs $0.13/kWh. Please find

(5a). (3p) The actual operating cost needed to maintain the ESB during a week (24/7)

**Solution:** Actual performance, consumption and cost:

\[
COP_{HP, act} = \gamma_{act} = 3.5 = \frac{\dot{Q}_H}{W_{in}} = \frac{\dot{\dot{Q}}_H}{\dot{W}_{in}} \quad \Rightarrow \quad \dot{W}_{in} = \frac{\dot{\dot{Q}}_H}{\gamma_{act}} = \frac{100 \text{ kW}}{3.5} = 28.57 \text{ kW}.
\]

The electrical energy consumed a week is

\[
W_{in} = \int \dot{W}_{in} dt = \dot{W}_{in} \Delta t = 28.57 \text{ kW} \times 168 \text{ hr} \approx 4,800 \text{ kWh} = 17,280 \text{ MJ},
\]

which costs \( 4,800 \text{ kWh} \times \$0.13/\text{kWh} = \$624 \).

(5b). (3p) The minimum theoretical weekly operating cost needed to maintain the ESB during a week

**Solution:** Maximum theoretical performance, associated with minimum theoretical consumption and costs

\[
COP_{HP,\, max, rev, cycle} = \gamma_{max} = \frac{T_H}{T_H - T_L} = \frac{300.15 \text{ K}}{25 \text{ K}} \approx 20 \quad \Rightarrow \quad \dot{W}_{in,\, \text{min}} = \frac{\dot{\dot{Q}}_H}{\gamma_{max}} = \frac{100 \text{ kW}}{20} = 5 \text{ kW}.
\]

The electrical energy consumed a day is

\[
W_{in} = \int \dot{W}_{in} dt = \dot{W}_{in} \Delta t = 5 \text{ kW} \times 168 \text{ hr} = 840 \text{ kWh} = 3,024 \text{ MJ},
\]

which yields the minimum theoretical costs of \( 840 \text{ kWh} \times \$0.13/\text{kWh} = \$109.2 \).

Alternatively, one can easily check that

\[
\frac{W_{in,\, \text{theor}}}{W_{in,\, \text{act}}} = \frac{\gamma_{act}}{\gamma_{max}} = \frac{3.5}{20} = 0.175 \Rightarrow W_{in,\, \text{theor}} = 0.175 \times W_{in,\, \text{act}}, \text{ costs } 0.175 \times \$624 = \$109.2.
\]

(5c). (4p) The potential cost to maintain the ESB during a week if the heat is provided by a resistor heater.

**Solution:** Potential performance, consumption and cost if the heat is provided by a resistor heater. In this case, for electric resistance heating, electricity provides all energy needed to achieve required heat transfer to the building. Namely,

\[
\dot{Q}_H = \dot{W}_{in} \quad \Rightarrow \quad W_{in} = \int \dot{Q}_H dt = \dot{Q}_H \Delta t = 100 \text{ kW} \times 168 \text{ hr} = 16,800 \text{ kWh} = 60,480 \text{ MJ},
\]

which costs \( 16,800 \text{ kWh} \times \$0.13/\text{kWh} = \$2184 \) >> than for a heat pump >> than for the ideal, reversible heat pump!

Alternatively, one can easily check that

\[
\frac{W_{in,\, \text{pot}}}{W_{in,\, \text{theor}}} = \frac{\gamma_{max}}{\gamma_{act}} = \frac{20}{3.5} = 5.71 \Rightarrow W_{in,\, \text{pot}} = 5.71 \times W_{in,\, \text{theor}}, \text{ costs } 5.71 \times \$624 = \$3,613.83.
\]

or

\[
\frac{W_{in,\, \text{pot}}}{W_{in,\, \text{act}}} = \frac{\gamma_{act}}{\gamma_{max}} = \frac{3.5}{20} = 0.175 \Rightarrow W_{in,\, \text{pot}} = 0.175 \times W_{in,\, \text{act}}, \text{ costs } 0.175 \times \$624 = \$109.2.
\]
6. (12 points) A piston-cylinder assembly, containing nitrogen, N\textsubscript{2}, undergoes a Carnot power cycle between two thermal reservoirs of temperatures 27 °C and 477 °C. After the isothermal expansion process, accompanied by the 55 kJ heat transfer to the gas, the volume and pressure of the system are 1 m\textsuperscript{3} and 0.6 MPa, respectively. Hint: you can assume N\textsubscript{2} to be an ideal gas at these conditions. Please determine:

(6a). (3p) the thermal efficiency of this power cycle;

**Solution**: 
\[ \eta_{th,Carnot} = \frac{T_H - T_L}{T_H} = \frac{(477 + 273.15) - (27 + 273.15)}{(477 + 273.15)} = \frac{450}{750.15} = 0.6 = 60\% . \]

(6b). (5p) the initial volume and pressure (those before the beginning of the isothermal expansion);

**Solution**: For an ideal gas, the internal energy is a function of temperature only. Since T\textsubscript{1} = T\textsubscript{2} = T\textsubscript{H}, we have \( u_1 = u_2 = u(T\textsubscript{H}) \), \( U_1 = U_2 = U(T\textsubscript{H}) \) \( \Rightarrow \Delta U = U_1 - U_2 = 0 \). On the other hand, the 1\textsuperscript{st} Law for process 1-2 reads: 
\[ \Delta U_{12} = Q_{12} - W_{12} \Rightarrow W_{12} = Q_{12} - \Delta U_{12} = Q_{12} - 0 = Q_{12} = 55 \text{ kJ} . \]

On the third hand, the boundary work \( W_{12} \):
\[ W_{12} = \int_{V_1}^{V_2} PdV = \frac{P_1}{\rho_1} \ln \left( \frac{V_2}{V_1} \right) = \frac{P_2}{\rho_2} \ln \left( \frac{V_2}{V_1} \right) \Rightarrow \ln \left( \frac{V_2}{V_1} \right) = \frac{W_{12}}{P_2 V_2} = \frac{55\text{kJ}}{600\text{kPa} \times 1\text{m}^3} = 0.092 \Rightarrow \]
\[ \frac{V_2}{V_1} = \exp(0.092) = 1.096 \Rightarrow V_1 = \frac{V_2}{1.096} = \frac{1\text{m}^3}{1.096} = 0.912\text{m}^3 . \]

Since \( T_1 = T_2 = T\textsubscript{H} \), ideal gas law yields: 
\[ P_1 V_1 = P_2 V_2 = mRT\textsubscript{H} \Rightarrow P_1 = \frac{V_2}{V_1} P_2 = 1.096 P_2 = 657.6 \text{ kPa} . \]

(6c). (2p) the heat rejected to cool reservoir;

**Solution**: 
\[ \eta_{th,Carnot} = 0.6 = \eta_{th} = 1 - \frac{Q_L}{Q_H} \Rightarrow Q_L = (1 - \eta_{th})Q_H = 0.4 Q_H = 0.4 Q_{12} = 0.4 \times 55 \text{kJ} = 22 \text{kJ} . \]

(6d). (2p) the work produced during this cycle.

**Solution**: From the 1\textsuperscript{st} Law:
\[ W_{cycle} = Q_{in} - Q_{out} = Q_{12} + Q_{34} = Q_H - Q_L = 55 \text{kJ} - 22 \text{kJ} = 33 \text{kJ} . \]

Alternatively, 
\[ \eta_{th} = \frac{W_{cycle}}{Q_H} \Rightarrow W_{cycle} = \eta_{th} Q_H = 0.6 \times 55 \text{kJ} = 33 \text{kJ} . \]
A heat pump with an average coefficient of performance of $\text{COP}_{\text{HP}} = 3.0$ heats the air in a rigid, insulated cuboid room of size $25 \text{ m} \times 10\text{ m} \times 4\text{ m}$. The heat pump consumes $15 \text{ kW}$ of power. The initial temperature and pressure in this room are $12 \degree \text{C}$ and 1 bar, respectively. How long will it take to raise the temperature in the room to $27 \degree \text{C}$?

**Solution:** With the density assumed to be constant, from the ideal gas law, the mass of the air in the room is

$$m = \frac{P_1 V_1}{RT_1} = \frac{100 \text{ kPa} \times (25 \times 10 \times 4) \text{ m}^3}{(0.287 \text{ kJ/kg} \cdot \text{K})(12 + 273.15) \text{K}} = 1222 \text{ kg}.$$

The specific heat at constant volume, $c_v$, of air at room temperature is $c_v = 0.718 \text{ kJ/kg} \cdot \text{C}$. The room is rigid => the boundary work over the process is zero ($dW = 0$). The system is closed => the 1st Law takes the form

$$Q_{12} = \Delta U_{12} + W_{12} = m(c_v)\Delta T = 1222 \text{ kg} \times 0.718 \frac{\text{kJ}}{\text{kg} \cdot \text{C}} \times (27 - 12) \degree \text{C} = 13,160 \text{ kJ}.$$

The electrical energy, consumed by the heat pump, is coupled to the heat transferred to the room and COP as

$$\text{COP}_{\text{HP}} = 3.0 = \frac{Q_H}{W_{\text{in}}} = \frac{Q_{12}}{W_{\text{in}}} \Rightarrow W_{\text{in}} = \frac{Q_{12}}{\gamma} = \frac{13,160 \text{ kJ}}{3} = 4,386.7 \text{ kJ}.$$

With the electrical power consumption $\dot{W}_{\text{in}} = 15 \text{ kW}$, this heating takes

$$\Delta t = \frac{W_{\text{in}}}{\dot{W}_{\text{in}}} = \frac{4,386.7 \text{ kJ}}{15 \text{ kJ/s}} = 292.4 \text{ s} \approx 5 \text{ min}.$$
(8). (13 points) The figure shows the main components of a steadily operating refrigerator using R-134a as a refrigerant. As shown in this figure, R134a enters the condenser at 800 kPa and 35 °C at a rate of 0.2 kg/s, and it leaves the condenser at 800 kPa as a saturated liquid. The compressor consumes 5.6 kW of power.

(8a). (7p) Please determine the rate of heat rejected from the condenser.

Solution: You can consider the condenser as a steady operation control volume system.

State 1: Inlet of the condenser: $P_{in} = 800$ kPa, $T_{in} = 35^\circ C$

The phase of R134a can be found by comparing the given temperature with the saturate temperature at given pressure:

$T_{sat@P_a} = 31.31^\circ C$

$T_{in} = 35^\circ C > T_{sat@P_a} = 31.31^\circ C$, superheated vapor.

For superheated vapor at $P_{in} = 800$ kPa, and $T_{in} = 35^\circ C$, you can find the specific enthalpy using interpolation:

$T_a = 31.31^\circ C$, $h_a = 267.29$ kJ/kg

$T_b = 40^\circ C$, $h_b = 276.45$ kJ/kg

$h_{in} = h_a + \frac{T_{in} - T_a}{T_b - T_a} (h_b - h_a) = 267.29 + \frac{35 - 31.31}{40 - 31.31} (276.45 - 267.29) = 271.18$ kJ/kg

State 2: Exit of condenser: $P_{out} = 800$ kPa, $x = 0$, saturated liquid;

$h_{out} = h_f@800$ kPa $= 95.47$ kJ/kg

You can find the heat transfer from the steady operating control volume system using the 1st law equation:

$\dot{Q} - \dot{W} + \dot{m}_{in} (h + \frac{1}{2} V^2 + gZ)_{in} - \dot{m}_{out} (h + \frac{1}{2} V^2 + gZ)_{out} = 0$

You can ignore the changes in work transfer and both potential and kinetic energy, the energy equation can be re-written to:

$\dot{Q} + \dot{m}_{in} h_{in} - \dot{m}_{out} h_{out} = \dot{Q} + m(h_{in} - h_{out}) = 0$

$= m(h_{out} - h_{in}) = 0.2$ kg/s $\times (95.47 - 271.18)$ kJ/kg $= -35.14$ kJ/s $= -35.14$ kW $= -\dot{Q}_H$.  

Note: $\dot{Q} < 0$ because this is the rate of heat rejected from this condenser. Hence, $\dot{Q}_H = |\dot{Q}| = 35.14$ kW.

(8b). (3p) Please determine the rate of heat absorption from the cooling compartment to the evaporator.

Solution: $\dot{Q}_L = \dot{Q}_H - \dot{W}_{in} = 35.14$ kW $- 5.6$ kW $= 29.54$ kW.

(8c). (3p) Please determine the (COP) of this refrigerator.

Solution: $COP_{RF} = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{29.54$ kW}{5.6$ kW} = 5.275.$