MAE 320–HW 7B

This comprehensive homework is due Monday, December 5th, 2016. Each problem is worth the points indicated. Copying of the solution from another is not acceptable.

1. Multi-choice, multi-answer question (16 Points)

1a. For ideal gas, please circle the property parameter (s) which is (are) functions of temperature only:

   (a) Specific internal energy \( u \)
   (b) Specific entropy \( s \)
   (c) Specific internal energy of ideal gas at ambient pressure: \( s^o \)
   (d) Relative pressure \( P_r \)
   (e) Relative specific volume \( v_r \)

1b. Please circle the items which are not state properties:

   (a) Total Internal Energy;
   (b) Heat Transfer;
   (c) Boundary work;
   (d) Entropy;
   (e) Entropy Generation;

1c. The transfer of entropy is usually associated with

   (a) The transfer of mechanical work such as shaft work;
   (b) The transfer of electrical work;
   (c) The transfer of heat;
   (d) The transfer of mass;
   (f) All above

1d. Please circle the cases satisfying the conservation of energy.

   (a) Closed system;
   (b) Isolated system;
   (c) Control volume system under steady operation;
   (d) Control volume under unsteady operation;
   (e) Any system and any process;
1e. For an isolated system, please circle the parameter which will change with time.

(a) **Total energy**;  
(b) **Total mass**;  
(c) Total entropy;  
(d) Entropy Generation;

1f. A heat engine cycle features with \( \int_{\text{Heat Engine}} \frac{dq}{T} < 0 \) operates between a hot reservoir at 1000 K and a cool reservoir at 300K. The efficiency of this heat engine?

(a) 100%;  
(b) 70%;  
(c) >70%;  
(e) <70%;  
(d) Cannot be decided with information provided;

1g. A heat engine cycle features with \( \int_{\text{Heat Engine}} \frac{dq}{T} = 0 \) operates between two thermal reservoirs. The statement “the thermal efficiency of this actual engine is 100%” violates:

(a) 1st law;  
(b) **The Kelvin-Plank Statement**;  
(c) **The Clausius Statement**;  
(c) **The Carnot Principle**;  
(d) **The Increase of Entropy Principle**;  
(e) Cannot be decided with information provided;

1h. For an actual cycle, please circle the valid equations

(a) \( \int dS = 0 \);  
(b) \( \int \frac{dq}{T} > 0 \);  
(c) \( \int \frac{dq}{T} = 0 \);  
(d) \( \int \frac{dq}{T} < 0 \);  
(e) \( S_{gen} < 0 \);
2. Short answer questions (24 Points)

2a. List all statements and principles associated with the 2nd law of thermodynamics. (2 points)

**Answers:**
1. The Kelvin-Planck Statement;
2. The Clausius Statement;
3. The Carnot Principle;
4. The increase of Entropy Principle;

2b. Please list at least three approaches capable increasing the entropy of a closed system. (3 points)

**Answer:**
1. the transfer of heat into the system;
2. the transfer of mass into the system;
3. the involvement of the irreversibility factors such as friction, chemical reaction, conversion from mechanical work to heat, the conversion from electrical work to heat, mixing of different species, etc.

2c. For ideal gas, please drive equation \(TP^{\frac{1-k}{k}} = C_1\) using isentropic process equation: \(Pv^k = C_2\). (3 points)

**Solution:** For ideal gas, \(Pv = RT\), so \(T = \frac{Pv}{R}\)

Load \(T = \frac{Pv}{R}\) to equation \(TP^{\frac{1-k}{k}} = C_1\), you can derive:

\[
\frac{Pv}{R} P^{\frac{1-k}{k}} = P^{\left(\frac{1}{1-k}\right)} \left(\frac{1}{k}\right) v = \frac{1}{R} = C_1
\]

It can be re-written as: \(P^k v = RC_1\)

\[
\left(\frac{1}{P^k v}\right)^k = \frac{1}{v} = \left(\frac{1}{RC_1}\right)^k = C_2
\]

You can derive: \(Pv^k = C_2\)

2d. The quality of energy is evaluated by the maximum percentage potential of such an energy to be converted to mechanical work. It is evident that the quality of energy is not affected by the
quantity of energy. Please list the following energy in the order of high quality to low one. (4 points):

(a) 1000 kJ electrical energy;
(b) 5000 kJ of thermal energy from hot reservoir at 600K;
(c) 100 kJ of thermal energy at 1000 K;
(d) 1 million kJ of thermal energy at 300 K.

If you need the temperature of the cool reservoir for the estimation of the maximum potential for thermal energy to mechanical energy conversion, you can assume the temperature of cool reservoir is 300 K.

Solution:

With the assumed cool reservoir temperature of 300 K, you can find the quality of the energy.

(1) the quality of electrical energy is 1 as it can be converted to mechanical work by 100%;

(2) The quality of thermal energy can be found by examining the maximum thermal efficiency of heat engine operating between the hot reservoir and cool reservoir:

$$
\eta_{HE,\text{max}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{600} = 0.5
$$

So the quality of thermal energy at 600 K is 0.5.

(3) $\eta_{HE,\text{max}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{1000} = 0.7$

(4) $\eta_{HE,\text{max}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{300} = 0$

Accordingly, the list of quality of energy should be (1) 1.0; (3) 0.7; (2) 0.5; (4) 0.0.

2e Find specific entropy of water at the following conditions and have them presented in the attached T-S diagram in next page: (6 points)

(a) $P = 10\text{bar}$, $T = 400^\circ\text{C}$;

(b) $T = 600^\circ\text{C}$, $h = 3600\text{kJ} / \text{kg}$;

(c) $P = 40\text{bar}$, $x = 0.3$

Solution:

(a) $P = 10\text{bar}$, $T = 400^\circ\text{C}$, superheated vapor; $s = 7.4670 \frac{kJ}{\text{kg} \cdot \text{K}}$

$$
s = 7.4 \sim 7.5 \frac{kJ}{\text{kg} \cdot \text{K}} \text{ (Chart A-9)},
$$

(b) $T = 600^\circ\text{C}$, $h = 3600\text{kJ} / \text{kg}$. It is difficult to find the $s$ using property table.
\[ s = 6.7 \sim 6.75 \frac{kJ}{kg \cdot K} \quad \text{(Chart A-9)} \]

(c) \[ P = 40\, \text{bar}, \; x = 0.3, \; s = s_y + x s_{fg} = 2.7966 + 0.3 \times 3.2731 = 3.7785 \frac{kJ}{kg \cdot K} \]

\[ s = 3.75 \sim 3.8 \frac{kJ}{kg \cdot K} \quad \text{(Chart A-9)} \]
2e. Water enters an adiabatic turbine at 100 bar and 700 °C (state 1) and exit at 100 kPa and 250 °C (state 2). Please mark state 1 and 2, 2s in the T-s diagram attached in next page and find the isentropic efficiency of this turbine. (6 Points)
**Solution:** Figure shows the state 1 (100 bar, 700°C), state 2a (100 kPa=1 bar, 250 °C), and state 2s (100 kPa).

\[
\begin{align*}
h_1 &= 3890 \frac{kJ}{kg} \; ; \; h_{2a} = 3000 \frac{kJ}{kg} \; ; \; h_{2s} = 2610 \frac{kJ}{kg}
\end{align*}
\]

The isentropic efficiency of this adiabatic turbine can be calculated:

\[
\eta_s = \frac{h_1 - h_{2s}}{h_1 - h_{2a}} = \frac{3890 - 3000}{3890 - 2610} = 69.53\%
\]
(3) A piston cylinder system initially contains 10 kg air at 40 bar and 800 K (state 1). After an isentropic expansion process \( P\nu^k = C \), the temperature drops to 600 K (state 2). Please determine: (12 Points)

(a) Find the average specific heat under constant volume and average specific heat ratio;

Solution: 
\[
T_{\text{avg}} = \frac{T_1 + T_2}{2} = \frac{800K + 600K}{2} = 700K
\]

\[
c_{c,\text{avg}} = 0.788 \times \frac{kJ}{kg \cdot K}, \quad k_{\text{avg}} = 1.364
\]

(b) Find the boundary work boundary work and heat transfer during this expansion process;

Solution:

Method (1): For isentropic process, \( P\nu^k = C \), which is a Polytropic process

\[
W_b = \frac{P_2V_2 - P_1V_1}{1 - k} = \frac{mR(T_2 - T_1)}{1 - k} = \frac{10kg \times 0.287 \times \frac{kJ}{kg \cdot K} (600K - 800K)}{1 - 1.364} = 1576.92kJ
\]

Method (2): You can also find pressure and volume at state 1 and state 2, respectively, and then find the boundary work:

Ideal gas: \( P\nu = RT \), so 
\[
v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 800K}{4000kPa} = 0.0574 m^3/kg
\]

\[
V_1 = mv_1 = 10kg \times 0.0574 m^3/kg = 0.564m^3
\]

\[
T_1P_1^{\frac{1}{k}} = T_2P_2^{\frac{1}{k}} = C, \quad P_2 = \left( \frac{T_1}{T_2} \right)^{\frac{k}{1-k}} P_1 = \left( \frac{800}{600} \right)^{\frac{1.364}{1-1.364}} \times 40\text{bar} = 13.628\text{bar} = 1362.8kPa
\]

\[
v_2 = \frac{RT_2}{P_2} = \frac{0.287 \times 600K}{1362.8kPa} = 0.1264 m^3/kg, \quad V_2 = mv_2 = 10kg \times 0.07297 m^3/kg = 1.264m^3
\]

The boundary work for the Polytropic system \( P\nu^k = C \) of ideal gas:

\[
W_b = \frac{P_2V_2 - P_1V_1}{1 - k} = \frac{(4000 \times 0.574 - 1362.8 \times 1.264)kPa \times m^3}{1 - 1.364} = 1575.33kJ
\]

Note: You can also find \( v_2 \) using equation \( T_1v_1^{k-1} = T_2v_2^{k-1} = C \),
\[
\frac{v_2}{v_1} = \left( \frac{T_1}{T_2} \right)^{\frac{1}{k-1}} = \left( \frac{800}{600} \right)^{\frac{1}{1.364-1}} = 2.204
\]

\[
v_2 = 2.204v_1 = 2.204 \times 0.0574 = 0.1265 \frac{m^3}{kg}
\]

(c) The change in the total entropy of air from state 1 and state 2;

**Solution:** For this polytropic process: \( T_1 P_1^{\frac{1-k}{k}} = T_2 P_2^{\frac{1-k}{k}} = C \),

\[
\frac{P_2}{P_1} = \left( \frac{T_1}{T_2} \right)^{\frac{k}{1-k}} = \left( \frac{800}{600} \right)^{\frac{1.364}{1-1.364}} = 0.34027
\]

\[
s_2 - s_1 = s_2^0 - s_1^0 - R \ln \frac{P_2}{P_1} = 2.40902 - 2.71787 - 0.287 \ln 0.3403 = 0.00054 \approx 0 \frac{kJ}{kg \cdot K}
\]

(d) The pressure of air at state 2 using equation \( TP^{\frac{1-k}{k}} = C \). You assume \( k=1.358 \).

**Solution:**

\[
T_1 P_1^{\frac{1-k}{k}} = T_2 P_2^{\frac{1-k}{k}} = C ,
\]

\[
P_2 = \left( \frac{T_1}{T_2} \right)^{\frac{k}{1-k}} P_1 = \left( \frac{800}{600} \right)^{\frac{1.358}{1-1.358}} \times 40bar = 13.432bar = 1343.2kPa
\]

(e) The pressure of air at state 2 using equation \( s_2 - s_1 = s_2^0 - s_1^0 - R \ln \frac{P_2}{P_1} \) or \( \frac{P_2}{P_1} = \frac{P_2}{P_{r1}} \) (are these two equations identical?)

**Solution:** For this isentropic process: \( s_2 - s_1 = s_2^0 - s_1^0 - R \ln \frac{P_2}{P_1} = 0 \)

\[
\frac{P_2}{P_1} = \exp \left( \frac{s_2^0 - s_1^0}{R} \right) = \exp \left( \frac{2.40902 - 2.71787}{0.287} \right) = 0.3409
\]

For this isentropic process:

\[
\frac{P_2}{P_1} = \frac{P_{r2}}{P_{r1}} = \frac{16.28}{47.75} = 0.3409
\]
(4). A rigid tank initially contains 5 kg air at 200 kPa and 127 °C (state 1). The air is gradually heated to 527 °C using thermal energy from hot reservoir at 1000 K (state 2). The blander installed in this tank consumed 400 kJ of mechanical work in maintaining the homogeneous temperature of the air within this tank. During this process, a heat loss of 600 kJ from this tank to ambient air occurs at boundary temperature 27 °C. Please determine (12 points)

(a) The pressure of air at state 2;

Solution: Ideal gas equation:

\[ \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \]

\[ P_2 = \frac{P_1 V_1 T_2}{V_2 T_1} = \frac{T_2}{T_1} P_1 = \frac{527 + 273}{127 + 273} \times 200kPa = 400kPa \]

(b) The change in total entropy of air during this process;

Solution: For ideal gas,

\[ S_2 - S_1 = s_2^o - s_1^o - R \ln \left( \frac{P_2}{P_1} \right) \]

\[ = 2.71787 - 1.99194 - 0.287 \ln \left( \frac{400kPa}{200kPa} \right) = 0.527 \frac{kJ}{kg \cdot K} \]

\[ S_2 - S_1 = m(s_2 - s_1) = 5\text{kg} \times 0.527 \frac{kJ}{kg \cdot K} = 2.635 \frac{kJ}{K} \]

(c) The entropy generation;

Solution: The heat transfer of this system includes a heat transfer of Q at 1000K and a heat loss of 600 kJ at 300 K. You find the heat transfer into the system prior to finding the entropy generation:

The 1st law of thermodynamics of rigid tank with shaft work but no boundary work:

\[ \sum Q - W = \Delta U = m \times (u_2 - u_1) \]

\[ \sum Q = Q_1 + Q_2 = W + m \times (u_2 - u_1) \], where \( Q_2 \) is the heat loss

\[ Q_1 = W + m \times (u_2 - u_1) - Q_2 = -400kJ + 5\text{kg}(592.30 - 301.6) \frac{kJ}{kg} - (-600kJ) = 1653.5kJ \]

Accordingly:

The entropy balance equation for this closed system:

\[ \sum \frac{Q}{T} + S_{gen} = \Delta S = m \times (s_2 - s_1) \]
\[
S_{\text{gen}} = m \times (s_2 - s_1) - \sum \frac{Q}{T} = m \times (s_2 - s_1) - \left( \frac{Q_1}{T_1} + \frac{Q_2}{T_2} \right)
\]

\[
= 5 \text{ kg} \times 0.527 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} - \left( \frac{1653.5 \text{ kJ}}{800 \text{ K}} + \frac{-600 \text{ kJ}}{300 \text{ K}} \right) = 2.7019 \frac{\text{kJ}}{\text{K}}
\]

Based on your answer to (c), is this process reversible, irreversible or impossible?

**Answer:** Since \( S_{\text{gen}} = 2.7019 \frac{\text{kJ}}{\text{K}} > 0 \), this process is irreversible.
(5) R134a enters an insulated throttling valve of an industry cooling unit as saturated liquid at 8 bar and exits at 80 kPa. The mass flow rate of this R-134a is 10 kg/s. Please find (12 points)

(a) The possible lowest temperature of refrigeration room for this industry cooling unit to work properly?

**Analysis:** The lowest temperature of the refrigeration room is the temperature of the coolant as the refrigerant is not able to remove heat from media with temperature lower than it. You need find the temperature of the refrigerant at exit of this insulated throttling valve.

**Solution:** For this insulated throttling valve:

\[ h_2 = h_1 = h_{f@800kPa} = 95.47 \frac{kJ}{kg} \]

\[ P_2 = 80kPa, \quad h_f = 11.21 \frac{kJ}{kg}, \quad h_g = 231.46 \frac{kJ}{kg}, \]

\[ h_f < h_2 = 95.47 \frac{kJ}{kg} < h_g \]

The phase at state 2 is saturated mixture at 80 kPa.

\[ T_2 = T_{f@804kPa} = -31.13^\circ C \], saturated mixture.

Accordingly, the possible lowest temperature of the refrigerant room is -31.13°C.

(b) The rate of entropy generation in this throttling valve.

**Solution:**

The specific entropy at state 1-saturated liquid at 8 bar=800 kPa

\[ s_1 = s_{f@800kPa} = 0.35404 \frac{kJ}{kg \cdot K} \]

The phase of R-137a is saturated mixture at 80 kPa and \( h_2 = 95.47 \frac{kJ}{kg} \). You can find \( s_2 \):

\[ \frac{h_2 - h_f}{h_{fg}} = \frac{s_2 - s_f}{s_{fg}} \]

\[ s_2 = s_f + \frac{h_2 - h_f}{h_{fg}} s_{fg} = 0.04711 + \frac{95.47 - 11.21}{220.25} \times 0.90999 = 0.3952 \frac{kJ}{kg \cdot K} \]

The entropy balance equation for this insulated throttling valve operating on steady-state:

\[ \frac{\dot{Q}}{T} + \dot{m}s_1 - \dot{m}s_2 + \dot{S}_{gen} = 0 \]
\[ \dot{S}_{\text{gen}} = \dot{m}(s_2 - \dot{m}s_1) - \frac{\dot{Q}}{T} = 10 \frac{\text{kg}}{\text{s}} \left(0.3952 - 0.35404\right) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} - \frac{0}{T} = 0.4116 \frac{\text{kJ}}{\text{K}} \]
(6). A steadily operated, insulated steam turbine is designed to produce 2626 kW of power. Water enters this turbine at 6 MPa, 600 °C (state 1) and leaves at 10 kPa (state 2). The mass flow rate of water is 2 kg/s. You can ignore the changes in the kinetic and the potential energy, please determine (12 points)

(a) The specific enthalpy, quality and specific entropy of water at exit (actual case);

Solution: You need two independent properties in determining the state of water. In this question, the pressure of water at exit was given. You need find another property parameter. Since the power of this steam turbine was known, you can find the specific entropy $s_2$ of water at exit.

1st law of thermodynamics for this steadily operating steam turbine

$$\dot{Q} - \dot{W} + \dot{m} h_1 - \dot{m} h_{2a} = 0$$

$$\dot{W} = \dot{m} h_1 - \dot{m} h_{2a} - \dot{Q} = \dot{m} h_1 - \dot{m} h_{2a}$$

$$h_{2a} = h_1 - \frac{\dot{W}}{\dot{m}} = 3658.8 - \frac{2626kW}{2kg/s} = 3658.8 - 1313 = 2345.8kJ/kg$$

State 2: $P_2 = 10kPa$, $h_{2a} = 2345.8kJ/kg$

$$h_{f@10kPa} = 191.81kJ/kg, \ h_{g@10kPa} = 2583.91kJ/kg$$

Since $h_f < h_2 < h_g$, the phase of water at state 2 is mixture of saturated liquid and vapor.

$$x_{2a} = \frac{h_{2a} - h_f}{h_g - h_f} = \frac{2345.8 - 191.81}{2583.9 - 191.81} = 0.90$$

$$s_{2a} = (1-x)s_f + xs_g = (1 - 0.9) \times 0.6492 + 0.9 \times 8.1488 = 7.3988\frac{kJ}{kg \cdot K}$$

(b) The rate of entropy generation within the turbine;

Solution: Entropy balance of this steam turbine

$$\sum \frac{\dot{Q}}{T} + \dot{m} s_1 - \dot{m} s_{2a} + \dot{S}_{gen} = \dot{m} s_1 - \dot{m} s_{2a} + \dot{S}_{gen} = 0$$: there is no heat transfer.

State 1: $P_1 = 6MPa$, $T_1 = 600°C$, superheated vapor, $s_1 = 7.1693\frac{kJ}{kg \cdot K}$

$$\dot{S}_{gen} = -\dot{m} s_1 + \dot{m} s_{2a} = \dot{m}(s_{2a} - s_1) = 2\frac{kg}{s}(8.0831 - 7.1693)\frac{kJ}{kg \cdot K} = 1.8276\frac{kJ}{K \cdot s}$$

$$\dot{S}_{gen} = -\dot{m} s_1 + \dot{m} s_{2a} - \frac{\dot{Q}}{T} = \dot{m}(s_{2a} - s_1) - \frac{\dot{Q}}{T} = 2\frac{kg}{s}(7.3988 - 7.1693)\frac{kJ}{kg \cdot K} - 0 = 0.459\frac{kJ}{K \cdot s}$$
(c) The isentropic efficiency of the turbine;

**Solution:** In order to calculate the isentropic efficiency: 

\[ \eta_{s, t} = \frac{h_1 - h_{2s}}{h_1 - h_{2a}} \]

you need find the specific enthalpy of water at exit \( h_{2s} \), if steam expanded following an isentropic process.

\[ s_{2s} = s_1 = 7.1693 \frac{kJ}{kg \cdot K}, \quad s_{s@10^5 Pa} = 0.6492 \frac{kJ}{kg \cdot K}, \quad s_{g@10^5 Pa} = 8.1488 \frac{kJ}{kg \cdot K} \]

Since \( s_f < s_{2s} < s_g \), the phase of water at exit after an isentropic expansion noted as \( 2s \) is mixture of saturated liquid and vapor.

\[ x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{s_1 - s_f}{s_g - s_f} = \frac{h_{2s} - h_f}{h_g - h_f} \]

\[ h_{2s} = h_f + \frac{s_{2s} - s_f}{s_g - s_f} \times (h_g - h_f) = 191.81 + \frac{7.1693 - 0.6492}{8.1488 - 0.6492} \times (2583.9 - 191.81) = 2271.5 \frac{kJ}{kg} \]

\[ \eta_{s, t} = \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{3658.8 - 2345.8}{3658.8 - 2271.5} = 0.946 = 94.6\% \]

(d) Present both the actual and isentropic processes in the \( T-s \) diagram attached in next page and have the initial state (1), the actual final state (2a), and the ideal final state (2s) clearly marked.

(e) Draw a schematic \( h-s \) diagram with both actual and the isentropic processes presented.
FIGURE A-9

T-s diagram for water.

(7) A piston cylinder system initially contains 10 kg of saturated liquid water at 200 kPa. The water was slowly heated under constant pressure until all of the liquid water was vaporized and became saturated vapor. During this process, the heat loss from the piston-cylinder system to ambient air is 5000 kJ, which occurs at its boundary with ambient air at 300 K. Please determine (12 Points):

a) The amount of heat transferred into the system during this process;

Solution:

State 1: \( P_1 = 200\text{kPa} \), saturated liquid, \( x_1 = 0 \)
\[ v_1 = v_{f\, @\, 200\text{kPa}} = 0.001061\text{m}^3 \]
\[ u_1 = u_{f\, @\, 200\text{kPa}} = 504.5\text{kJ} / \text{kg} \]
\[ s_1 = s_{f\, @\, 200\text{kPa}} = 1.5302\text{kJ} / (\text{kg} \cdot \text{K}) \]

State 2: \( P_2 = 200\text{kPa} \), saturated vapor, \( x_2 = 1 \)
\[ v_2 = v_{g\, @\, 200\text{kPa}} = 0.88578\text{m}^3 \]
\[ u_2 = u_{f\, @\, 200\text{kPa}} = 2529.1\text{kJ} / \text{kg} \]
\[ s_2 = s_{f\, @\, 200\text{kPa}} = 7.1270\text{kJ} / (\text{kg} \cdot \text{K}) \]

1st law of thermodynamics
\[ \sum Q - W_b = Q_1 + Q_2 + \Delta U = m(u_2 - u_1) \]
\[ Q_1 = -Q_2 + W_b + m(u_2 - u_1) = -Q_2 + P(v_2 - v_1) + m(u_2 - u_1) = -Q_2 + Pm(v_2 - v_1) + m(u_2 - u_1) \]
\[ = -(-5000\text{kJ}) + 200\text{kPa} \times 10\text{kg} \times (0.88578 - 0.001061)\text{m}^3 / \text{kg} + 10\text{kg}(2529.1 - 504.5)\frac{\text{kJ}}{\text{kg}} \]
\[ = 27015.4\text{kJ} \]

b) The total entropy generated if the heat needed to heat this system was provided by a hot reservoir with a temperature of 600 K;

Solution: If heat needed is provided by a hot reservoir, there are totally two heat transfer involved: (a) from hot reservoir to water in cylinder; (b) heat transfer from water in cylinder to ambient air;

\[ \sum \frac{Q}{T} + S_{gen} = \Delta S = m \times (s_2 - s_1) \]
\[ S_{gen} = \Delta S = m \times (s_2 - s_1) - \sum \frac{Q}{T} \]
\[ = 10\text{kg} \times (7.1270 - 1.5302)\frac{\text{kJ}}{\text{kg} \cdot \text{K}} - 27015.4\frac{\text{kJ}}{600\text{K}} - \frac{-5000\text{kJ}}{300\text{K}} \]
\[ = 27.605 \frac{\text{kJ}}{\text{K}} \]
c) The total entropy generated if the heat was provided by a resister heater with the consumption of electricity;

**Solution:** If heat needed is provided by a resister heater, there is only one heat transfer involved: heat transfer from water in cylinder to ambient air;

\[
\frac{Q}{T} + S_{gen} = \Delta S = m \times (s_2 - s_1)
\]

\[
S_{gen} = \Delta S = m \times (s_2 - s_1) - \frac{Q}{T}
\]

\[
= 10\text{kg} \times (7.1270 - 1.5302) \frac{kJ}{\text{kg} \cdot K} - \frac{-5000 kJ}{300 K}
\]

\[
= 72.6 \frac{kJ}{K}
\]

d) Explain why more entropy was generated when the heat was provided by a resister heater?

**Answer:** The increased entropy with resister heater was due to the increase in entropy of the resister heater during the conversion from electricity to heat.
Practice Questions. The solutions to these questions will be posted with other questions. You are encouraged to solve these questions although your answers to these two questions will not be graded.

(1) A resistor heater installed in a hot water tank is used to steadily heat water from 10 °C to 60 °C at a mass flow rate of 12 kg/min. The heat loss from water tank to ambient air is 20 kW, which occurs at boundary temperature of 17 °C. You can assume the operation pressure is 100 kPa, and ignore the changes in both kinetic energy and potential energy. The specific enthalpy of compressed liquid water can be calculated using equation $h_{p,T} = h_{f@T}$. Please find the power of the resistor heater and rate of entropy generation in this hot water tank.

**Solution:** Figure shows the schematic diagram of the hot water tank.

1st law of thermodynamic for steady operation control volume system

\[
\dot{Q} - \dot{W} + \dot{m}h_1 - \dot{m}h_2 = 0
\]

\[
\dot{W} = \dot{Q} + \dot{m}h_1 - \dot{m}h_2 = \dot{Q} + \dot{m}(h_1 - h_2) = \dot{Q} + \dot{m}(h_{f@10^\circ C} - h_{f@60^\circ C})
\]

\[
= -20\text{ kW} + 12 \frac{\text{kg}}{\text{min}} \times \frac{\text{min}}{60\text{s}} \times (42.022 - 251.18)\text{ kJ/kg}
\]

\[
= -61.83 \frac{\text{kJ}}{\text{s}} = -61.83\text{kW}
\]

The entropy generation can be found using the balance of entropy of control volume system

\[
\frac{\dot{Q}}{T} + \dot{m}s_1 - \dot{m}s_2 + \dot{S}_{\text{gen}} = 0
\]

\[
\dot{S}_{\text{gen}} = -\frac{\dot{Q}}{T} - \dot{m}s_1 + \dot{m}s_2 = -\frac{\dot{Q}}{T} + \dot{m}(s_2 - s_1)
\]

\[
= -\frac{-20\text{kW}}{273 + 17} + 0.2\text{ kg/s} \times (0.8313 - 0.1511) \frac{kJ}{\text{kg} \cdot \text{K}} = 0.205 \frac{kJ}{\text{K} \cdot \text{s}}
\]
(2) A well insulated compressor operated under steady state. The CO₂ enters this compressor at 100 kPa and 25°C with a mass flow rate of 10 kg/s and exits at 5 bar and 450 K. The rate of heat loss from compressor to ambient air is 30 kW occurring at 300 K. The changes in kinetic and potential energy can be neglected. Please determine

(a) The actual work input of this compressor;

**Solution:**

State 1: \( P_1 = 100kPa , T_1 = 25°C = 298K \)

\[
h_1 = \frac{\bar{h}_1}{M_{CO_2}} = \frac{9364kJ \text{ / kmol}}{44kg \text{ / kmol}} = 212.82kJ \text{ / kg}
\]

\( \bar{s}_1^o = 213.685kJ \text{ / kg} \)

State 2a: \( P_2 = 5bar = 500kPa , T_{2a} = 450K \)

\[
h_{2a} = \frac{\bar{h}_1}{M_{CO_2}} = \frac{15483kJ \text{ / kmol}}{44kg \text{ / kmol}} = 351.89kJ \text{ / kg}
\]

\( \bar{s}_{2a}^o = 230.194kJ \text{ / kg} \)

The 1st law of thermodynamics:

\[
\dot{Q} - \dot{W} + \dot{m}h_1 - \dot{m}h_{2a} = 0
\]

\[
\dot{W} = \dot{Q} + \dot{m}h_1 - \dot{m}h_{2a} = \dot{Q} + \dot{m}(h_1 - h_{2a})
\]

\[
= -30kW + 10 \text{ kg/s} \times (212.82 - 351.89) \frac{kJ}{kg} = -1420.7kW
\]

(b) The rate of entropy generation in this compressor;

**Solution:** The entropy balance equation:

\[
\frac{\dot{Q}}{T} + \dot{m}s_1 - \dot{m}s_{2a} + \dot{S}_{gen} - \frac{\dot{Q}}{T} + \dot{m}(s_1 - s_{2a}) = \dot{S}_{gen} = 0
\]

\[
\dot{S}_{gen} = -\frac{\dot{Q}}{T} + \dot{m}(s_{2a} - s_1) = -\frac{\dot{Q}}{T} + \dot{m}\left(s_{2a}^o - s_1^o - R \ln \frac{P_2}{P_1}\right) = -\frac{\dot{Q}}{T} + \dot{m}\left(\frac{-s_{2a}^o - s_1^o - R \ln \frac{P_2}{P_1}}{M_{CO_2}}\right)
\]

\[
= -30kW \div 300K + 10kg/s \times \left(\frac{(230.194 - 213.685 - 8.314 \ln 5)}{44kg \text{ / kmol}} \frac{kJ}{kmol \cdot K}\right) = 0.81kW \div K
\]
(c) Based on your answer to (b), is this compression process internally reversible, irreversible or impossible;

**Answer:** As \( \dot{S}_{\text{gen}} \) is positive, the compression process is an irreversible process.

(d) The exit temperature of CO\(_2\) if compressed to 5 bar following an isentropic process;

**Solution:** For an isentropic compression process:

\[
\bar{s}_{2s} - \bar{s}_1 = \bar{s}_2' - \bar{s}_1' - R \ln \frac{P_2}{P_1} = 0 \\
\bar{s}_{2s}' = \bar{s}_1' + R \ln \frac{P_2}{P_1} = 213.685 + 8.314 \ln 5 = 227.066 \frac{kJ}{\text{kmol} \cdot K}
\]

With the known \( \bar{s}_{2s}' = 227.066 \frac{kJ}{\text{kmol} \cdot K} \), you can find the temperature using interpolation:

\[
T_{2s} = 410K + \frac{227.066 - 226.250}{227.258 - 226.250} \times 10K = 418.09K
\]

(e) The isentropic efficiency of this compressor;

**Solution:** With the known \( \bar{s}_{2s}' = 227.066 \frac{kJ}{\text{kmol} \cdot K} \), you can find the specific enthalpy of CO\(_2\) at exit after isentropic compression process:

\[
\bar{h}_{2s} = 13787 + \frac{227.066 - 226.250}{227.258 - 226.250} \times (14206 - 13787) = 14126.19 \frac{kJ}{\text{kmol} \cdot K}
\]

\[
h_{2s} = \frac{\bar{h}_{2s}}{M_{\text{CO2}}} = \frac{14126.19}{44} = 321.05 \frac{kJ}{\text{kg} \cdot K}
\]

The isentropic efficiency of this compressor can be calculated:

\[
\eta_{s,c} = \frac{h_{2s} - h_1}{h_{2a} - h_1} = \frac{321.05 - 212.82}{351.89 - 212.82} = 77.82\%
\]